2019-Ph-H2-Q4

Section: Our Dynamic Universe
Topic: Gravitation

Summary:
A communications satellite orbits at a height of $36.0 \times 10^6\,\mathrm{m}$ above Earth's surface.
Given:

- Mass of Earth: $M = 6.0 \times 10^{24}\,\mathrm{kg}$,
- Radius of Earth: $R_E = 6.4 \times 10^6\,\mathrm{m}$,
- Gravitational force on the satellite: $F = 57\,\mathrm{N}$.

Tasks:
(a) Find the distance between the centre of the Earth and the satellite.
(b) Calculate the mass of the satellite.
(c) Determine the gravitational field strength g at the satellite.
(d) Compare gravitational forces for a second satellite (quarter mass, half distance).

Solution:

(a) Distance from Earth's centre:
$r = R_E + \text{height} = 6.4 \times 10^6 + 36.0 \times 10^6 = 42.4 \times 10^6\,\mathrm{m}.$

(b) Mass of the satellite:
Gravitational force:

$$F = G\frac{Mm}{r^2} \Rightarrow m = \frac{Fr^2}{GM}.$$

Substitute:

$$m = \frac{57\,(42.4 \times 10^6)^2}{6.67 \times 10^{-11} \times 6.0 \times 10^{24}}.$$

$$m \approx \frac{57 \times 1.80 \times 10^{15}}{4.00 \times 10^{14}} \approx 256\,\mathrm{kg}.$$

Answer: $m \approx 2.6 \times 10^2\,\mathrm{kg}.$

(c) Gravitational field strength at the satellite:

$$g = \frac{F}{m} = \frac{57}{256} \approx 0.22\,\mathrm{N\,kg^{-1}}.$$

(d) Second satellite comparison:
Gravitational force:

$$F \propto \frac{m}{r^2}.$$

- Mass is $\frac{1}{4}m$.
- Distance is $\frac{1}{2}r$, so $\frac{1}{r^2}$ increases by 4.

Thus:

$$F_2 = \frac{1}{4}m \cdot 4\frac{1}{r^2} = F.$$

Answer: The gravitational force is the same as for the first satellite.

Guidance for Students:

- Always add Earth's radius to altitude for orbital distance.
- Use the universal law of gravitation $F = G\frac{Mm}{r^2}$.
- Gravitational field strength g can be found from $g = F/m$ or $g = GM/r^2$.

Revision Tips:

- Remember: $G = 6.67 \times 10^{-11}\,\mathrm{Nm^2\,kg^{-2}}$.
- Carefully handle large exponents (scientific notation).
- For proportionality problems, compare each factor (mass, distance) step by step.