(9) (a) $y = \frac{1}{3}x^3 - x^2 - 3x + 1$

$\frac{dy}{dx} = x^2 - 2x - 3$

SPs when $\frac{dy}{dx} = 0$:

$x^2 - 2x - 3 = 0$

$(x+1)(x-3) = 0$

$x+1 = 0$ or $x-3 = 0$

$x = -1$ or $x = 3$

When $x = -1$, $y = \frac{1}{3}(-1)^3 - (-1)^2 - 3(-1) + 1$

$= -\frac{1}{3} - 1 + 3 + 1$

$= \frac{8}{3}$ $\left(-1, \frac{8}{3}\right)$

When $x = 3$, $y = \frac{1}{3}(3^3) - 3^2 - 3(3) + 1$

$= 9 - 9 - 9 + 1$

$= -8$ $(3, -8)$

(b) When $x = 6$, $y = \frac{1}{3}(6^3) - 6^2 - 3(6) + 1$

$= 72 - 36 - 18 + 1$

$= 19$

x	-1	3	6
y	$\frac{8}{3}$	-8	19

19 ← Greatest value = 19

-8 ↑ Least value = -8

Question			Generic scheme	Illustrative scheme	Max mark
9.	(a)		$\bullet^1$ differentiate two non-constant terms $\bullet^2$ complete derivative and equate to 0 $\bullet^3$ find x-coordinates $\bullet^4$ find y-coordinates	$\bullet^1$ eg x^2-2x $\bullet^2$ $x^2-2x-3=0$ $\bullet^3$ $-1,\quad 3$ $\bullet^4$ $\frac{8}{3}$, -8	**4**

Notes:

1. For a numerical approach, award 0/4.
2. $\bullet^2$ is only available if '$=0$' appears at the $\bullet^2$ stage or in working leading to $\bullet^3$. However, see Candidate A.
3. $\bullet^3$ is only available for solving a quadratic equation.
4. $\bullet^3$ and $\bullet^4$ may be awarded vertically.

Commonly Observed Responses:

Candidate A

Stationary points when $\frac{dy}{dx}=0$

$\frac{dy}{dx}=x^2-2x-3$ $\bullet^1$ ✓ $\bullet^2$ ✓

$\frac{dy}{dx}=(x+1)(x-3)$

$x=-1,\ 3$ $\bullet^3$ ✓

$y=\frac{8}{3},\ -8$ $\bullet^4$ ✓

Candidate B - derivative never equated to 0

x^2-2x-3 $\bullet^1$ ✓ $\bullet^2$ ^

$(x+1)(x-3)$

$x=-1,\ 3$ $\bullet^3$ ✓$_1$

$y=\frac{8}{3},\ -8$ $\bullet^4$ ✓

	(b)		$\bullet^5$ evaluate y at $x=6$ $\bullet^6$ state greatest and least values	$\bullet^5$ 19 $\bullet^6$ greatest = 19 and least = -8	**2**

Notes:

5. 'Greatest $(6,19)$; least $(3,-8)$' does not gain $\bullet^6$.
6. Where $x=-1$ was not identified as a stationary point in part (a), y must also be evaluated at $x=-1$ to gain $\bullet^6$.
7. $\bullet^6$ is not available for using y at a value of x, obtained at $\bullet^3$ stage, which lies outwith the interval $-1\le x\le 6$.
8. $\bullet^6$ is only available where candidates have attempted to evaluate y at $x=6$.

Commonly Observed Responses: