(7) "Upper − lower" $= (6+4x-2x^2)-(x^3-6x^2+11x)$

$$= 6+4x-2x^2-x^3+6x^2-11x$$

$$= -x^3+4x^2-7x+6$$

$$\text{Area} = \int_0^2 (-x^3+4x^2-7x+6)\,dx$$

$$= \left[-\frac{x^4}{4}+\frac{4x^3}{3}-\frac{7x^2}{2}+6x\right]_0^2$$

$$= \left(-\frac{2^4}{4}+\frac{4(2^3)}{3}-\frac{7(2^2)}{2}+6(2)\right)-(0)$$

$$= -\frac{16}{4}+\frac{32}{3}-\frac{28}{2}+12$$

$$= -4+\frac{32}{3}-14+12$$

$= \frac{14}{3}$ square units

Question			Generic scheme	Illustrative scheme	Max mark
7.			**Method 1**	**Method 1**	5
			•[1] integrate using 'upper' – 'lower'	•[1] $\int\left(\left(6+4x-2x^2\right)-\left(x^3-6x^2+11x\right)\right)dx$	
			•[2] identify limits	•[2] $\int_0^2\left(\left(6+4x-2x^2\right)-\left(x^3-6x^2+11x\right)\right)dx$	
			•[3] integrate	•[3] $6x-\frac{7}{2}x^2+\frac{4}{3}x^3-\frac{1}{4}x^4$	
			•[4] substitute limits	•[4] $\left(6(2)-\frac{7}{2}(2)^2+\frac{4}{3}(2)^3-\frac{1}{4}(2)^4\right)-0$	
			•[5] evaluate area	•[5] $\frac{14}{3}$ (units^2)	
			Method 2	**Method 2**	
			•[1] know to integrate between appropriate limits for both equations	•[1] $\int_0^2 \ldots dx$ and $\int_0^2 \ldots dx$	
			•[2] integrate both functions	•[2] $6x+\frac{4x^2}{2}-\frac{2x^3}{3}$ and $\frac{x^4}{4}-\frac{6x^3}{3}+\frac{11x^2}{2}$	
			•[3] substitute limits into both expressions	•[3] $\left(6(2)+\frac{4(2)^2}{2}-\frac{2(2)^3}{3}\right)-0$ and $\left(\frac{(2)^4}{4}-\frac{6(2)^3}{3}+\frac{11(2)^2}{2}\right)-0$	
			•[4] evaluate both integrals	•[4] $\frac{44}{3}$ and 10	
			•[5] evidence of subtracting areas	•[5] $\frac{14}{3}$ (units^2)	

Question	Generic scheme	Illustrative scheme	Max mark

7. (continued)

Notes:

1. Correct answer with no working - award 1/5.
2. Do not penalise lack of 'dx' at •1 in Method 1.
3. In Method 1, limits and 'dx' must appear by the •2 stage for •2 to be awarded and in Method 2 by the •1 stage for •1 to be awarded.
4. In Method 1, treat the absence of brackets at •1 stage as bad form only if the correct integrand is obtained. See Candidates C and D.
5. Where a candidate differentiates one or more terms, or fails to integrate, no further marks are available.
6. In Method 1, accept unsimplified expressions such as $6x+\frac{4x^2}{2}-\frac{2x^3}{3}-\frac{x^4}{4}+\frac{6x^3}{3}-\frac{11x^2}{2}$ at •3.
7. Do not penalise the inclusion of '$+c$'.
8. Do not penalise the continued appearance of the integral sign or dx after integrating.
9. •5 is not available where solutions include statements such as '$-\frac{14}{3}=\frac{14}{3}$ square units'. See Candidates A and B.
10. In Method 1, where a candidate uses an invalid strategy the only marks available are •3 for integrating a polynomial with at least four terms (of different degree) and •4 for substituting the limits of 0 and 2 into the resulting expression. However, see Candidate E.
11. At •4, do not penalise candidates for who reduce powers of 0. For example writing 0 in place of 0^4. Similarly, do not penalise candidates writing 0 in place of $6(0)$. However, candidates who write 0^3 in place of 0^4 or $2(0)$ in place of $6(0)$ do not gain •4.

Commonly Observed Responses:

Candidate A - switched limits		Candidate B - 'lower' - 'upper'	
$\int_2^0 \left(\left(6+4x-2x^2\right)-\left(x^3-6x^2+11x\right)\right)dx$	•2 ✓	$\int_0^2 \left(\left(x^3-6x^2+11x\right)-\left(6+4x-2x^2\right)\right)dx$	•2 ✓
		$\int_0^2 x^3-4x^2+7x-6\,dx$	
$=6x-\frac{7}{2}x^2+\frac{4}{3}x^3-\frac{1}{4}x^4$	•3 ✓	$=\frac{1}{4}x^4-\frac{4}{3}x^3+\frac{7}{2}x^2-6x$	•3 ✓
$=0-\left(6(2)-\frac{7}{2}(2)^2+\frac{4}{3}(2)^3-\frac{1}{4}(2)^4\right)$	•4 ✓	$\left(\frac{1}{4}(2)^4-\frac{4}{3}(2)^3+\frac{7}{2}(2)^2-6(2)\right)-(0)$	•4 ✓
$=-\frac{14}{3}$		$=-\frac{14}{3}$	
$=\frac{14}{3}$	•1 ✗ •5 ✗	$\therefore$ Area $=\frac{14}{3}$	•1 ✓ •5 ✓

Question	Generic scheme	Illustrative scheme	Max mark

7. (continued)

Candidate C - missing brackets

$$\int_0^2 6+4x-2x^2-x^3-6x^2+11x\,dx$$

$$\int_0^2 6-7x+4x^2-x^3\,dx$$ $\bullet^1$ ✓ $\bullet^2$ ✓

Candidate D - missing brackets

$$\int_0^2 6+4x-2x^2-x^3-6x^2+11x\,dx$$ $\bullet^1$ ✗ $\bullet^2$ ✓$_1$

$$\int_0^2 6+15x-8x^2-x^3\,dx$$

$$6x+\frac{15}{2}x^2-\frac{8}{3}x^3-\frac{1}{4}x^4$$ $\bullet^3$ ✓$_1$

$$\left(6(2)+\frac{15}{2}(2)^2-\frac{8}{3}(2)^3-\frac{1}{4}(2)^4\right)-(0)$$ $\bullet^4$ ✓$_1$

$$\frac{50}{3}$$ $\bullet^5$ ✓$_1$

Candidate E - 'upper' + 'lower'

$$\int_0^2\left(\left(6+4x-2x^2\right)+\left(x^3-6x^2+11x\right)\right)dx$$ $\bullet^1$ ✗ $\bullet^2$ ✓$_1$

$$6x+\frac{15}{2}x^2-\frac{8}{3}x^3+\frac{1}{4}x^4$$ $\bullet^3$ ✓$_1$

$$\left(6(2)+\frac{15}{2}(2)^2-\frac{8}{3}(2)^3+\frac{1}{4}(2)^4\right)-0$$ $\bullet^4$ ✓$_1$

$$\frac{74}{3}$$ $\bullet^5$ ✓$_1$

Candidate F - incorrect substitution

$$\int_0^2\left(\left(6+4x-2x^2\right)-\left(x^3-6x^2+11x\right)\right)dx$$ $\bullet^1$ ✓ $\bullet^2$ ✓

$$\left(6x+2x^2-\frac{2}{3}x^3\right)-\left(\frac{1}{4}x^4-2x^3+\frac{11}{2}x^2\right)$$ $\bullet^3$ ✓

$$\left(6(2)+2(2)^2-\frac{2}{3}(2)^3\right)-\left(\frac{1}{4}(0)^4-2(0)^3+\frac{11}{2}(0)^2\right)$$ $\bullet^4$ ✗

$$\frac{44}{3}$$ $\bullet^5$ ✓$_2$