③ (a) $\vec{ED} = \vec{OD} - \vec{OE} = \begin{pmatrix} 2 \\ -3 \\ 4 \end{pmatrix} - \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix} = \begin{pmatrix} 1 \\ -4 \\ 6 \end{pmatrix}$

$\vec{EF} = \vec{OF} - \vec{OE} = \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} - \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$

(b) (i) $\vec{ED} \cdot \vec{EF} = \begin{pmatrix} 1 \\ -4 \\ 6 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} = 2 + (-4) + 18 = 16$

(ii) $|\vec{ED}| = \sqrt{1^2 + (-4)^2 + 6^2} = \sqrt{1+16+36} = \sqrt{53}$

$|\vec{EF}| = \sqrt{2^2 + 1^2 + 3^2} = \sqrt{4+1+9} = \sqrt{14}$

$$\vec{ED} \cdot \vec{EF} = |\vec{ED}||\vec{EF}| \cos\theta$$

$$16 = \sqrt{53}\sqrt{14} \cos\theta$$

$$\cos\theta = \frac{16}{\sqrt{53}\sqrt{14}}$$

$$\theta = \cos^{-1}\left(\frac{16}{\sqrt{53}\sqrt{14}}\right)$$

$$= 54.0288\ldots$$

$$\approx 54.0^\circ$$

| Question | | | Generic scheme | Illustrative scheme | Max mark |
|---|---|---|---|---|---|
| **3.** | (a) | | •[1] find $\overrightarrow{ED}$<br><br>•[2] find $\overrightarrow{EF}$ | •[1] $\begin{pmatrix} 1 \\ -4 \\ 6 \end{pmatrix}$<br><br>•[2] $\begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$ | **2** |
| **Notes:** | | | | | |
| 1. For candidates who find **both** $\overrightarrow{DE}$ **and** $\overrightarrow{FE}$ correctly, award 1/2.<br>2. Accept vectors written horizontally. | | | | | |
| **Commonly Observed Responses:** | | | | | |
| | | | | | |
| | (b) | (i) | •[3] evaluate $\overrightarrow{ED}.\overrightarrow{EF}$ | •[3] 16 | **1** |
| | | (ii) | •[4] evaluate $\lvert\overrightarrow{ED}\rvert$<br><br>•[5] evaluate $\lvert\overrightarrow{EF}\rvert$<br><br>•[6] substitute into formula for scalar product<br><br>•[7] calculate angle | •[4] $\sqrt{53}$<br><br>•[5] $\sqrt{14}$<br><br>•[6] $\cos DEF = \frac{16}{\sqrt{53}\times\sqrt{14}}$ or<br>$\sqrt{53}\times\sqrt{14}\times\cos DEF = 16$<br><br>•[7] 54.028...° or<br>0.942... radians | **4** |

| Question | Generic scheme | Illustrative scheme | Max mark |
|---|---|---|---|

**3. (b) (continued)**

**Notes:**

3. Do not penalise candidates who treat negative signs with a lack of rigour when calculating a magnitude. For example accept $\sqrt{1^2+4^2+6^2}=\sqrt{53}$ or $\sqrt{1^2+-4^2+6^2}=\sqrt{53}$ for $\bullet^4$.
   However, do not accept $\sqrt{1^2-4^2+6^2}=\sqrt{53}$ for $\bullet^4$.
4. $\bullet^6$ is not available to candidates who simply state the formula $\cos\theta=\dfrac{\overrightarrow{ED}.\overrightarrow{EF}}{|\overrightarrow{ED}||\overrightarrow{EF}|}$.
   However, $\cos\theta=\dfrac{16}{\sqrt{53}\times\sqrt{14}}$ and $\sqrt{53}\times\sqrt{14}\times\cos\theta=16$ are acceptable for $\bullet^6$.
5. Accept correct answers rounded to 54° or 0.9 radians (or 60 gradians).
6. Do not penalise the omission or incorrect use of units.
7. $\bullet^7$ is only available as a result of using a valid strategy.
8. $\bullet^7$ is only available for a single angle.
9. For a correct answer with no working award 0/4.

**Commonly Observed Responses:**

| Candidate A - poor notation | | Candidate B - insufficient communication | |
|---|---|---|---|
| $\begin{pmatrix}1\\-4\\6\end{pmatrix}\cdot\begin{pmatrix}2\\1\\3\end{pmatrix}=\begin{pmatrix}2\\-4\\18\end{pmatrix}=16$ | $\bullet^3$ ✗ | $\lvert\overrightarrow{ED}\rvert=\sqrt{53}$ | $\bullet^4$ ✓ |
| | | $\lvert\overrightarrow{EF}\rvert=\sqrt{14}$ | $\bullet^5$ ✓ |
| | | $\dfrac{16}{\sqrt{53}\times\sqrt{14}}$ | $\bullet^6$ ^ |
| | | 54.028…° or 0.942… radians | $\bullet^7$ ✓$_1$ |
| **Candidate C - BEWARE** | | | |
| $\lvert\overrightarrow{OF}\rvert=\sqrt{14}$ | $\bullet^5$ ✗ | | |