

(12)

$$2 \sin 2x - \sin^2 x = 0$$

$$2(2 \sin x \cos x) - \sin^2 x = 0$$

$$4 \sin x \cos x - \sin^2 x = 0$$

$$(\sin x)(4 \cos x - \sin x) = 0$$

$$\sin x = 0 \quad \text{or} \quad 4 \cos x - \sin x = 0$$

$$\underline{x = 0^\circ, 180^\circ} \quad \text{or} \quad \sin x = 4 \cos x$$

$$\frac{\sin x}{\cos x} = 4$$

$$\tan x = 4$$

$$\begin{aligned} \text{Related acute angle} &= \tan^{-1} 4 \\ &= 75.963\dots \\ &\approx 76.0^\circ \end{aligned}$$

$$x = 76.0^\circ, 180 + 76.0^\circ$$

$$\underline{x = 76.0^\circ, 256.0^\circ}$$

S	A ✓
I	C

Question			Generic scheme	Illustrative scheme	Max mark
12.			<p>•<sup>1</sup> substitute appropriate double angle formula</p> <p>•<sup>2</sup> factorise</p> <p>•<sup>3</sup> solve for <math>\tan x^\circ</math></p> <p>•<sup>4</sup> solve <math>\tan x^\circ = 4</math></p> <p>•<sup>5</sup> solve <math>\sin x^\circ = 0</math></p>	<p>•<sup>1</sup> <math>2(2 \sin x^\circ \cos x^\circ) - \sin^2 x^\circ (= 0)</math></p> <p>•<sup>2</sup> <math>\sin x^\circ (4 \cos x^\circ - \sin x^\circ) = 0</math></p> <p>•<sup>3</sup> <math>\tan x^\circ = 4</math> (since <math>x = 90, 270</math> are not solutions)</p> <p>•<sup>4</sup> <math>76, 256</math></p> <p>•<sup>5</sup> <math>0, 180</math></p>	5

#### Notes:

- <sup>1</sup> is still available to candidates who correctly substitute for  $\sin^2 x$  in addition to  $\sin 2x$ .
- Substituting  $2 \sin A \cos A$  for  $\sin 2x^\circ$  at the •<sup>1</sup> stage should be treated as bad form provided the equation is written in terms of  $x$  at the •<sup>2</sup> stage. Otherwise, •<sup>1</sup> is not available.
- '= 0' must appear by the •<sup>2</sup> stage for •<sup>2</sup> to be awarded.
- Award •<sup>2</sup> for ' $S(4C - S) = 0$ ' only where  $\sin x^\circ = 0$  and  $4 \cos x^\circ - \sin x^\circ = 0$  appear.
- Do not penalise the omission of degree signs.
- At •<sup>3</sup> stage, candidates are not required to check that 90 and 270 are not solutions before dividing by  $\cos x^\circ$ . Where candidates have divided by  $\sin x$  at the •<sup>2</sup> stage without considering  $\sin x = 0$ , •<sup>3</sup> and •<sup>4</sup> are still available.
- At •<sup>3</sup> stage, candidates may use the wave function and arrive at  $\sqrt{17} \cos(x + 14)^\circ = 0$ , or an equivalent wave form, instead of  $\tan x^\circ = 4$ .
- <sup>4</sup> is only available where the working at the •<sup>3</sup> stage is of equivalent difficulty to reaching  $\tan x^\circ = 4$ .
- <sup>5</sup> is not available where  $\sin x = 0$  comes from an invalid strategy.
- For candidates who work only in radians, •<sup>5</sup> is not available.
- <sup>4</sup> and •<sup>5</sup> may be awarded vertically. See also Candidate B.
- Do not penalise solutions outwith  $0 \leq x < 360$ .

#### Commonly Observed Responses:

<p><b>Candidate A - working in radians</b></p> <p>∴</p> <p><math>\tan x^\circ = 4</math></p> <p>1.326, 4.468</p> <p><math>0, \pi</math></p>	<p>•<sup>1</sup> ✓   •<sup>2</sup> ✓</p> <p>•<sup>3</sup> ✓</p> <p>•<sup>4</sup> ✓<sub>1</sub></p> <p>•<sup>5</sup> ✓<sub>2</sub></p>	<p><b>Candidate B - partial solutions</b></p> <p><math>2(2 \sin x^\circ \cos x^\circ) - \sin^2 x^\circ = 0</math></p> <p><math>\sin x^\circ (4 \cos x^\circ - \sin x^\circ) = 0</math></p> <p><math>\sin x^\circ = 0</math></p> <p><math>x = 180</math></p> <p><math>\tan x^\circ = 4</math></p> <p><math>x = 76</math></p> <p>•<sup>5</sup> ^</p>	<p>•<sup>1</sup> ✓</p> <p>•<sup>2</sup> ✓</p> <p>•<sup>3</sup> ✓</p> <p>•<sup>4</sup> ✓</p>
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