(11) (a) $k\cos(x-a) = k\cos x \cos a + k\sin x \sin a$

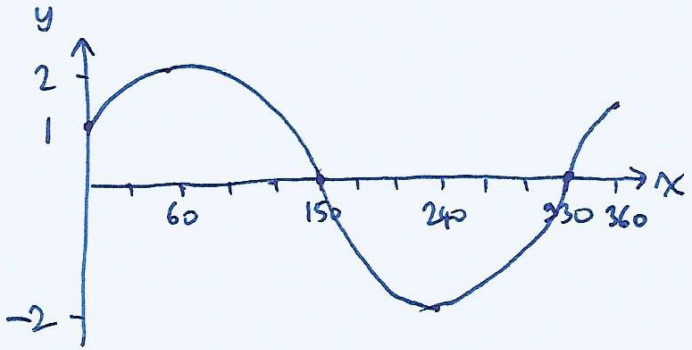
$$\left.\begin{aligned} k\cos a &= 1 \\ k\sin a &= \sqrt{3} \end{aligned}\right\} \quad \tan a = \sqrt{3}$$

$$a = 60^\circ$$

$$k^2 = 1^2 + (\sqrt{3})^2$$
$$= 1 + 3$$
$$= 4$$
$$k = 2$$

$$2\cos(x-60)^\circ$$

(b)



y-intercept:

$y = 2\cos(0-60)^\circ$

$= 2\cos 300^\circ$

$= 2\cos 60^\circ$

$= 2 \times \frac{1}{2}$

$= 1$

S	A
T	C

Question			Generic scheme	Illustrative scheme	Max mark
11.	(a)		$\bullet^1$ use compound angle formula $\bullet^2$ compare coefficients $\bullet^3$ process for k $\bullet^4$ process for a and express in required form	$\bullet^1$ $k\cos x°\cos a°+k\sin x°\sin a°$ **stated explicitly** $\bullet^2$ $k\cos a°=1, k\sin a°=\sqrt{3}$ **stated explicitly** $\bullet^3$ $k=2$ $\bullet^4$ $2\cos(x-60)°$	4

Notes:

1. Accept $k(\cos x°\cos a°+\sin x°\sin a°)$ for $\bullet^1$. Treat $k\cos x°\cos a°+\sin x°\sin a°$ as bad form only if the equations at the $\bullet^2$ stage both contain k.
2. Do not penalise the omission of degree signs.
3. $2\cos x°\cos a°+2\sin x°\sin a°$ or $2(\cos x°\cos a°+\sin x°\sin a°)$ is acceptable for $\bullet^1$ and $\bullet^3$.
4. $\bullet^2$ is not available for $k\cos x°=1, k\sin x°=\sqrt{3}$, however $\bullet^4$ may still be gained- see Candidate E
5. $\bullet^3$ is only available for a single value of $k, k>0$.
6. $\bullet^3$ is not available to candidates who work with $\sqrt{4}$ throughout parts (a) and (b) without explicitly simplifying at any stage. $\bullet^4$ is still available.
7. $\bullet^4$ is not available for a value of a given in radians.
8. Candidates may use any form of the wave function for $\bullet^1$, $\bullet^2$and $\bullet^3$. However, $\bullet^4$ is only available if the wave is interpreted in the form $k\cos(x-a)°$.
9. Evidence for $\bullet^4$ may not appear until part (b).

Commonly Observed Responses:

Candidate A

$\bullet^1$ ^

$2\cos a°=1$

$2\sin a°=\sqrt{3}$ $\bullet^2$ ✓ $\bullet^3$ ✓

$\tan a°=\sqrt{3}$

$a=60$

$2\cos(x-60)°$ $\bullet^4$ ✓

Candidate B - inconsistency

$k\cos x°\cos a°+k\sin x°\sin a°$ $\bullet^1$ ✓

$\cos a°=1$

$\sin a°=\sqrt{3}$ $\bullet^2$ ✗

$\tan a°=\sqrt{3}$

$a=60$

$2\cos(x-60)°$ $\bullet^3$ ✓ $\bullet^4$ ✗

Candidate C

$\cos x°\cos a°+\sin x°\sin a°$ $\bullet^1$ ✗

$\cos a°=1$

$\sin a°=\sqrt{3}$ $\bullet^2$ ✓2

$k=2$ $\bullet^3$ ✓

$\tan a°=\sqrt{3}$

$a=60$

$2\cos(x-60)°$ $\bullet^4$ ✗

Question	Generic scheme	Illustrative scheme	Max mark
11. (a) (continued)			

Candidate D - errors at $\bullet^2$

$k\cos x°\cos a° + k\sin x°\sin a°$ $\bullet^1$✓

$k\cos a° = \sqrt{3}$

$k\sin a° = 1$ $\bullet^2$ ✗

$\tan a° = \dfrac{1}{\sqrt{3}}$

$a = 30$

$2\cos(x-30)°$ $\bullet^3$✓ $\bullet^4$✓$_1$

Candidate E - use of x at $\bullet^2$

$k\cos x°\cos a° + k\sin x°\sin a°$ $\bullet^1$✓

$k\cos x° = 1$

$k\sin x° = \sqrt{3}$ $\bullet^2$ ✗

$\tan x° = \sqrt{3}$

$x = 60$

$2\cos(x-60)°$ $\bullet^3$✓ $\bullet^4$✓$_1$

Candidate F

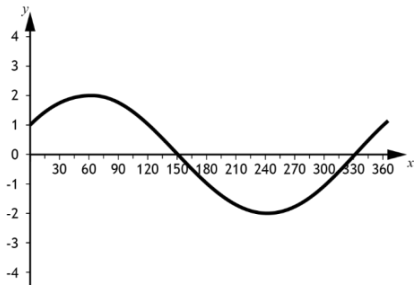
$k\sin A\cos B + k\cos A\sin B$ $\bullet^1$ ✗

$k\cos A = 1$

$k\sin A = \sqrt{3}$ $\bullet^2$ ✗

$\tan A = \sqrt{3}$

$2\cos(x-60)°$ $\bullet^3$✓ $\bullet^4$✓$_1$

Question			Generic scheme	Illustrative scheme	Max mark
11.	(b)		$\bullet^5$ exactly two roots identifiable from graph $\bullet^6$ coordinates of exactly two turning points identifiable from graph $\bullet^7$ y-intercept and value of y at $x = 360$ identifiable from graph	$\bullet^5$ $(150,0)$ and $(330,0)$ $\bullet^6$ $(60,2)$ and $(240,-2)$ $\bullet^7$ 1 	**3**

Notes:

10. $\bullet^5$, $\bullet^6$ and $\bullet^7$ are only available for attempting to draw a "cosine" graph with a period of $360°$.
11. Ignore any part of a graph drawn outwith $0 \le x \le 360$.
12. Vertical marking is not applicable to $\bullet^5$ and $\bullet^6$.
13. Candidate's sketch in (b) must be consistent with the equation obtained in (a), see also Candidates G and H.
14. For any incorrect horizontal translation of the graph of the wave function arrived at in part (a) only $\bullet^6$ is available.

Commonly Observed Responses:

Candidate G – incorrect translation

(a) $2\cos(x-60)°$ - correct equation

(b) Incorrect translation:
Sketch of $2\cos(x+60)°$
only $\bullet^6$ is available

Candidate H – incorrect equation

(a) $2\cos(x+60)°$ - incorrect equation

(b) Sketch of $2\cos(x+60)°$
all 3 marks available