(10) (a)

1	2	3	−4	−3	2
	↓	2	5	1	−2
	2	5	1	−2	0

Remainder = 0

So $(x-1)$ is a factor.

So $2x^4+3x^3-4x^2-3x+2 = (x-1)(2x^3+5x^2+x-2)$

(b) Try $x=1$. $2+5+1-2 \neq 0$

Try $x=-1$. $-2+5-1-2=0$

−1	2	5	1	−2
	↓	−2	−3	2
	2	3	−2	0

$(x-1)(x+1)(2x^2+3x-2)$

$=(x-1)(x+1)(x+2)(2x-1)$

Question			Generic scheme	Illustrative scheme	Max mark
10.	(a)		$\bullet^1$ use 1 in synthetic division or in evaluation of quartic	$\bullet^1$ 1 | 2 3 −4 −3 2 2 | □ **or** $2\times(1)^4+3\times(1)^3-4\times(1)^2-3\times(1)+2$	**2**
			$\bullet^2$ complete division/evaluation and interpret result	$\bullet^2$ 1 | 2 3 −4 −3 2 2 5 1 −2 2 5 1 −2 | 0 Remainder $=0\therefore(x-1)$ is a factor **or** $f(1)=0\therefore(x-1)$ is a factor	

Notes:

1. Communication at $\bullet^2$ must be consistent with working at that stage i.e. a candidate's working must arrive legitimately at 0 before $\bullet^2$ can be awarded.
2. Accept any of the following for $\bullet^2$:
   - '$f(1)=0$ so $(x-1)$ is a factor'
   - 'since remainder $=0$, it is a factor'
   - the '0' from any method linked to the word 'factor' by 'so', 'hence', $\therefore$, $\rightarrow$, $\Rightarrow$ etc.
3. Do not accept any of the following for $\bullet^2$:
   - double underlining the '0' or boxing the '0' without comment
   - '$x=1$ is a factor', '... is a root'
   - the word 'factor' only, with no link.

Commonly Observed Responses:

Candidate A - grid method

	$2x^3$			
x	$2x^4$	$5x^3$		
-1	$-2x^3$			

$\bullet^1$ ✓

	$2x^3$	$5x^2$	x	-2
x	$2x^4$	$5x^3$	x^2	$-2x$
-1	$-2x^3$	$-5x^2$	$-x$	2

'with no remainder'

$\therefore(x-1)$ is a factor $\bullet^2$ ✓

Candidate B - grid method

	$2x^3$			
x	$2x^4$	$5x^3$		
-1	$-2x^3$			

$\bullet^1$ ✓

	$2x^3$	$5x^2$	x	-2
x	$2x^4$	$5x^3$	x^2	$-2x$
-1	$-2x^3$	$-5x^2$	$-x$	2

$\therefore(x-1)(2x^3+5x^2+x-2)=2x^4+3x^3-4x^2-3x+2$

$\therefore(x-1)$ is a factor $\bullet^2$ ✓

Question			Generic scheme	Illustrative scheme	Max mark
10.	(b)		$\bullet^3$ identify cubic and attempt to factorise	$\bullet^3$ eg $-1 \mid 2 \quad 5 \quad 1 \quad -2$ $\quad\quad -2 \quad -3$ $\quad\quad 2 \quad 3 \quad \square$ **or** $-2 \mid 2 \quad 5 \quad 1 \quad -2$ $\quad\quad -4 \quad -2$ $\quad\quad 2 \quad 1 \quad \square$	**4**
			$\bullet^4$ find second factor	$\bullet^4$ eg $-1 \mid 2 \quad 5 \quad 1 \quad -2$ $\quad\quad -2 \quad -3 \quad 2$ $\quad\quad 2 \quad 3 \quad -2 \quad 0$ leading to $(x+1)$ **or** $-2 \mid 2 \quad 5 \quad 1 \quad -2$ $\quad\quad -4 \quad -2 \quad 2$ $\quad\quad 2 \quad 1 \quad -1 \quad 0$ leading to $(x+2)$	
			$\bullet^5$ identify quadratic	$\bullet^5$ $2x^2+3x-2$ **or** $2x^2+x-1$	
			$\bullet^6$ complete factorisation	$\bullet^6$ $(x-1)(x+1)(2x-1)(x+2)$ **stated explicitly**	

Notes:

4. Ignore the appearance of '$=0$'.
5. Candidates who arrive at $(x-1)(x+1)(2x^2+3x-2)$ or $(x-1)(x+2)(2x^2+x-1)$ by using algebraic long division or by inspection, gain $\bullet^3$, $\bullet^4$ and $\bullet^5$.
6. Where a candidate only identifies additional factors from a quartic, only $\bullet^4$ is available.
7. $\bullet^3$ and $\bullet^4$ may be awarded for applications of synthetic division even when previous trials contain errors. $\bullet^5$ and $\bullet^6$ are available.

Question	Generic scheme	Illustrative scheme	Max mark

10. (b) (continued)

Commonly Observed Responses:

Candidate C - grid method

(a)

	$2x^3$	$5x^2$	x	-2
x	$2x^4$	$5x^3$	x^2	$-2x$
-1	$-2x^3$	$-5x^2$	$-x$	2

(b)

	$2x^2$	…	…
x	$2x^3$	…	…
…	…	…	…

$\bullet^3$ ✓

$\bullet^3$ is awarded for evidence of the cubic expression (which may be in the grid from part (a)) **AND** the terms in the diagonal boxes summing to the second and third terms in the cubic respectively.

	$2x^2$	$3x$	-2
x	$2x^3$	$3x^2$	$-2x$
$+1$	$2x^2$	$3x$	-2

$\bullet^4$ ✓

$2x^2+3x-2$ $\bullet^5$ ✓

$(x-1)(x+1)(2x-1)(x+2)$ $\bullet^6$ ✓

Candidate D - grid method

(a)

	$2x^3$	$5x^2$	x	-2
x	$2x^4$	$5x^3$	x^2	$-2x$
-1	$-2x^3$	$-5x^2$	$-x$	2

(b)

	$2x^2$	…	…
x	$2x^3$	…	…
…	…	…	…

$\bullet^3$ ✓

$\bullet^3$ is awarded for evidence of the cubic expression (which may be in the grid from part (a)) **AND** the terms in the diagonal boxes summing to the second and third terms in the cubic respectively.

	$2x^2$	x	-1
x	$2x^3$	x^2	$-x$
$+2$	$4x^2$	$2x$	-2

$\bullet^4$ ✓

$2x^2+x-1$ $\bullet^5$ ✓

$(x-1)(x+2)(x+1)(2x-1)$ $\bullet^6$ ✓

Candidate E

$\frac{1}{2}$	2	5	1	-2
		1	3	2
	2	6	4	0

$\bullet^3$ ✓ $\bullet^4$ ✓

$\left(x-\frac{1}{2}\right)\left(2x^2+6x+4\right)$ $\bullet^5$ ✓

$(2x-1)\left(x^2+3x+2\right)$

$(x-1)(2x-1)(x+1)(x+2)$ $\bullet^6$ ✓

Candidate F

$\frac{1}{2}$	2	5	1	-2
		1	3	2
	2	6	4	0

$\bullet^3$ ✓ $\bullet^4$ ✓

$\left(x-\frac{1}{2}\right)\left(2x^2+6x+4\right)$ $\bullet^5$ ✓

$\left(x-\frac{1}{2}\right)(2x+2)(x+2)$

$(x-1)\left(x-\frac{1}{2}\right)(2x+2)(x+2)$ $\bullet^6$ ^