

9.(a)

$$\begin{aligned} k \sin(x+a) &= k \sin x \cos a + k \cos x \sin a \\ &= k \cos a \sin x + k \sin a \cos x \\ &= k \sin a \cos x + k \cos a \sin x \\ &7 \cos x - 3 \sin x \end{aligned}$$

$$k \sin a = 7$$

$$k \cos a = -3$$

$$k = \sqrt{7^2 + 3^2}$$

$$k = \sqrt{58}$$

$$\tan a = \frac{7}{-3}$$

$$a = 113$$

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$$\tan^{-1}(a) = 67$$

$$7 \cos x - 3 \sin x = \sqrt{58} \sin(x+113)^\circ$$

9.(b)
(i)

$$\begin{aligned} &14 \cos x - 6 \sin x \\ &= 2(7 \cos x - 3 \sin x) \\ &= 2\sqrt{58} \sin(x+113)^\circ \end{aligned}$$

$$\begin{aligned} \text{Max value} \\ &= 2\sqrt{58} \end{aligned}$$

9.(b)
(ii)

$$x+113 = 90, 450$$

$$x = 337^\circ$$

$$0 \leq x < 360$$

$$113 \leq x+113 < 473$$

Question			Generic scheme	Illustrative scheme	Max mark
9.	(a)		<ul style="list-style-type: none"> •¹ use compound angle formula •² compare coefficients •³ process for k •⁴ process for a and express in required form 	<ul style="list-style-type: none"> •¹ $k \sin x^\circ \cos a^\circ + k \cos x^\circ \sin a^\circ$ stated explicitly •² $k \cos a^\circ = -3, k \sin a^\circ = 7$ stated explicitly •³ $\sqrt{58}$ •⁴ $\sqrt{58} \sin(x + 113.19\dots)^\circ$. 	4

Notes:

1. Do not penalise the omission of degree symbols in this question.
2. Accept $k(\sin x^\circ \cos a^\circ + \cos x^\circ \sin a^\circ)$ at •¹.
3. Treat $k \sin x^\circ \cos a^\circ + \cos x^\circ \sin a^\circ$ as bad form only if the equations at the •² stage both contain k .
4. $\sqrt{58} \sin x^\circ \cos a^\circ + \sqrt{58} \cos x^\circ \sin a^\circ$ or $\sqrt{58}(\sin x^\circ \cos a^\circ + \cos x^\circ \sin a^\circ)$ are acceptable for •¹ and •³.
5. •² is not available for $k \cos x^\circ = -3$ and $k \sin x^\circ = 7$, however •⁴ may still be gained - see Candidate E.
6. •³ is only available for a single value of $k, k > 0$.
7. •⁴ is not available for a value of a given in radians.
8. Accept values of a which round to 113.
9. Candidates may use any form of the wave function for •¹, •² and •³. However, •⁴ is only available if the wave is interpreted in the form $k \sin(x + a)^\circ$.
10. Evidence for •⁴ may appear in part (b).

Question	Generic scheme	Illustrative scheme	Max mark
9. (continued)			
Commonly Observed Responses:			
Candidate A $\sqrt{58} \cos a^\circ = -3$ $\sqrt{58} \sin a^\circ = 7$ $\tan a^\circ = -\frac{7}{3}$ $a = 113.19\dots$ $\sqrt{58} \sin(x + 113.19\dots)^\circ$	Candidate B $k \sin x^\circ \cos a^\circ + k \cos x^\circ \sin a^\circ$ $\cos a^\circ = -3$ $\sin a^\circ = 7$ $\tan a^\circ = -\frac{7}{3}$ $a = 113.19\dots$ $\sqrt{58} \sin(x + 113.19\dots)^\circ$ <div>Not consistent with equations at \bullet^2.</div>	Candidate C $\sin x^\circ \cos a^\circ + \cos x^\circ \sin a^\circ$ $\cos a^\circ = -3$ $\sin a^\circ = 7$ $k = \sqrt{58}$ $\tan a^\circ = -\frac{7}{3}$ $a = 113.19\dots$ $\sqrt{58} \sin(x + 113.19\dots)^\circ$	$\bullet^1 \wedge$ $\bullet^2 \checkmark \bullet^3 \checkmark$ $\bullet^4 \checkmark$ $\bullet^1 \times$ $\bullet^2 \boxed{\checkmark}_2$ $\bullet^3 \checkmark$ $\bullet^4 \times$
Candidate D - errors at \bullet^2 $k \sin x \cos a + k \cos x \sin a$ $k \cos a^\circ = 7$ $k \sin a^\circ = -3$ $\tan a^\circ = -\frac{3}{7}$ $a = 336.80\dots$ $\sqrt{58} \sin(x + 336.80\dots)^\circ$	Candidate E - use of x at \bullet^2 $k \sin x \cos a + k \cos x \sin a$ $k \cos x^\circ = -3$ $k \sin x^\circ = 7$ $\tan a^\circ = -\frac{7}{3}$ $a = 113.19\dots$ $\sqrt{58} \sin(x + 113.19\dots)^\circ$	Candidate F $k \sin A \cos B + k \cos A \sin B$ $k \cos A = -3$ $k \sin A = 7$ $\tan A = -\frac{7}{3}$ $A = 113.19\dots$ $\sqrt{58} \sin(x + 113.19\dots)^\circ$	$\bullet^1 \checkmark$ $\bullet^2 \times$ $\bullet^3 \checkmark \bullet^4 \boxed{\checkmark}_1$ $\bullet^1 \times$ $\bullet^2 \times$ $\bullet^3 \checkmark \bullet^4 \boxed{\checkmark}_1$ $\bullet^1 \times$ $\bullet^2 \times$ $\bullet^3 \checkmark \bullet^4 \boxed{\checkmark}_1$

Question			Generic scheme	Illustrative scheme	Max mark
9.	(b)	(i)	• ⁵ state maximum value	• ⁵ $2\sqrt{58}$	1
		(ii)	<p>Method 1</p> <p>•⁶ start to solve</p> <p>•⁷ state value of x</p> <p>Method 2</p> <p>•⁶ start to solve</p> <p>•⁷ state value of x</p>	<p>Method 1</p> <p>•⁶ $x + 113.19... = 90$ leading to $x = -23.19...$</p> <p>•⁷ $x = 336.80...$</p> <p>Method 2</p> <p>•⁶ $x + 113.19... = 450$</p> <p>•⁷ $x = 336.80...$</p>	2
Notes:					
11. • ⁷ is only available where an angle outwith the range $0 \leq x < 360$ needs to be considered - see Candidate G.					
12. • ⁷ is only available where • ⁶ has been awarded. However, see Candidate K.					
Commonly Observed Responses:					
Candidate G - not considering angle outwith $0 \leq x < 360$ $\sqrt{58} \sin(x - 23)^\circ$ from part (a) $x - 23 = 90$ $x = 113$			• ⁶ <input checked="" type="checkbox"/> 1 • ⁷ <input checked="" type="checkbox"/> 2	Candidate H - simplification (i) $2\sqrt{58}$ (ii) $\sqrt{58} \sin(x + 113)^\circ = \sqrt{58}$ $x + 113 = 90$ $x = -23$ $x = 337$	• ⁵ ✓ • ⁶ ✓ • ⁷ ✓
Candidate I - follow-through marking (i) $\sqrt{58}$ (ii) $2\sqrt{58} \sin(x + 113)^\circ = \sqrt{58}$ $x + 113 = 30$ $x = -83$ $x = 277$			• ⁵ ✗ • ⁶ <input checked="" type="checkbox"/> 1 • ⁷ <input checked="" type="checkbox"/> 1	Candidate J - graphical approach (i) $\sqrt{58}$ (ii) max occurs when $x + 113 = 90$ $x = -23$ $x = 337$	• ⁵ ✗ • ⁶ ✓ • ⁷ ✓
Candidate K - no acknowledgement of $\times 2$ (i) $\sqrt{58}$ (ii) $\sqrt{58} \sin(x + 113)^\circ = \sqrt{58}$ $x + 113 = 90$ $x = -23$ $x = 337$			• ⁵ ✗ • ⁶ ✗ • ⁷ <input checked="" type="checkbox"/> 1		