QUESTION NUMBER

DO NOT WRITE IN THIS MARGIN

8.

$$\int_{-2}^{1} (x^3 - 2x^2 - 4x + 1) - (x - 5)\, dx$$

$$= \int_{-2}^{1} x^3 - 2x^2 - 5x + 6 \, dx$$

$$= \left[\frac{x^4}{4} - \frac{2x^3}{3} - \frac{5x^2}{2} + 6x \right]_{-2}^{1}$$

$$= \left[\frac{1^4}{4} - \frac{2(1)^3}{3} - \frac{5(1)^2}{2} + 6(1) \right]$$

$$- \left[\frac{(-2)^4}{4} - \frac{2(-2)^3}{3} - \frac{5(-2)^2}{2} + 6(-2) \right]$$

$$= \frac{37}{12} - \left(-\frac{38}{3} \right)$$

$$= \frac{63}{4} \text{ square units}$$

Question				Generic scheme	Illustrative scheme	Max mark
8.				**Method 1**	**Method 1**	5
				•¹ integrate using "upper - lower"	•¹ $\int\left(\left(x^3-2x^2-4x+1\right)-\left(x-5\right)\right)dx$	
				•² identify limits	•² $\int_{-2}^{1}\left(\left(x^3-2x^2-4x+1\right)-\left(x-5\right)\right)dx$	
				•³ integrate	•³ $\frac{x^4}{4}-\frac{2x^3}{3}-\frac{5x^2}{2}+6x$	
				•⁴ substitute limits	•⁴ $\left(\frac{(1)^4}{4}-\frac{2(1)^3}{3}-\frac{5(1)^2}{2}+6(1)\right)-\left(\frac{(-2)^4}{4}-\frac{2(-2)^3}{3}-\frac{5(-2)^2}{2}+6(-2)\right)$	
				•⁵ calculate shaded area	•⁵ $\frac{63}{4}$ or $15\frac{3}{4}$	
				Method 2	**Method 2**	5
				•¹ know to integrate between appropriate limits for both integrals	•¹ $\int_{-2}^{1}\ldots dx$ and $\int_{-2}^{1}\ldots dx$	
				•² integrate both functions	•² $\frac{x^4}{4}-\frac{2x^3}{3}-\frac{4x^2}{2}+x$ and $\frac{x^2}{2}-5x$	
				•³ substitute limits into both expressions	•³ $\left(\frac{(1)^4}{4}-\frac{2(1)^3}{3}-\frac{4(1)^2}{2}+(1)\right)-\left(\frac{(-2)^4}{4}-\frac{2(-2)^3}{3}-\frac{4(-2)^2}{2}+(-2)\right)$ **and** $\left(\frac{(1)^2}{2}-5(1)\right)-\left(\frac{(-2)^2}{2}-5(-2)\right)$	
				•⁴ evaluate both integrals	•⁴ $-\frac{3}{4}$ and $-\frac{33}{2}$	
				•⁵ evidence of subtracting areas	•⁵ $-\frac{3}{4}-\left(-\frac{33}{2}\right)=\frac{63}{4}$	

Question		Generic scheme	Illustrative scheme	Max mark
8.	**(continued)**			

Notes:

1. Correct answer with no working – award 1/5.
2. In Method 1, treat the absence of brackets at $\bullet^1$ stage as bad form only if the correct integral is obtained at $\bullet^3$ - see Candidates A and B.
3. Do not penalise lack of 'dx' at $\bullet^1$.
4. Limits and 'dx' must appear by the $\bullet^2$ stage for $\bullet^2$ to be awarded in Method 1 and by the $\bullet^1$ stage for $\bullet^1$ to be awarded in Method 2.
5. Where a candidate differentiates one or more terms at $\bullet^3$, then $\bullet^3$, $\bullet^4$ and $\bullet^5$ are unavailable.
6. Accept unsimplified expressions at $\bullet^3$ e.g. $\frac{x^4}{4}-\frac{2x^3}{3}-\frac{4x^2}{2}+x-\frac{x^2}{2}+5x$.
7. Do not penalise the inclusion of '$+c$'.
8. Do not penalise the continued appearance of the integral sign after $\bullet^2$
9. Candidates who substitute limits without integrating do not gain $\bullet^3$, $\bullet^4$ or $\bullet^5$.
10. $\bullet^5$ is not available where solutions include statements such as '$-\frac{63}{4}=\frac{63}{4}$ square units' - see Candidate B.
11. Where a candidate only integrates x^3-2x^2-4x+1 or another cubic or quartic expression, only $\bullet^3$ and $\bullet^4$ are available (from Method 1).

Question	Generic scheme	Illustrative scheme	Max mark
8. (continued)			

Commonly Observed Responses:

Candidate A - bad form corrected

$\int_{-2}^{1} x^3-2x^2-4x+1-x-5\,dx$ •2 ✓

$=\frac{x^4}{4}-\frac{2x^3}{3}-\frac{5x^2}{2}+6x$ •3 ✓ ⇒ •1 ✓

Bad form at •1 must be corrected by the integration stage and may also take the form of a missing minus sign

Candidate B

$\int_{-2}^{1} x^3-2x^2-4x+1-x-5\,dx$ •1 ✗ •2 ✓

$=\frac{x^4}{4}-\frac{2x^3}{3}-\frac{5x^2}{2}-4x$ •3 ✓1

$=\left(\frac{(1)^4}{4}-\frac{2(1)^3}{3}-\frac{5(1)^2}{2}-4(1)\right)$

$-\left(\frac{(-2)^4}{4}-\frac{2(-2)^3}{3}-\frac{5(-2)^2}{2}-4(-2)\right)$ •4 ✓1

$-\frac{57}{4}$ cannot be negative so $=\frac{57}{4}$ •5 ✗

However, $\int\ldots=-\frac{57}{4}$ so Area $=\frac{57}{4}$ •5 ✓1

Candidate C - lower – upper

$\int_{-2}^{1}\left((x-5)-(x^3-2x^2-4x+1)\right)dx$ •2 ✓

$-\frac{x^4}{4}+\frac{2x^3}{3}+\frac{5x^2}{2}-6x$ •3 ✓

$\left(-\frac{(1)^4}{4}+\frac{2(1)^3}{3}+\frac{5(1)^2}{2}-6(1)\right)-$

$\left(-\frac{(-2)^4}{4}+\frac{2(-2)^3}{3}+\frac{5(-2)^2}{2}-6(-2)\right)$ •4 ✓

$-\frac{63}{4}$

So Area $=\frac{63}{4}$ •1 ✓ •5 ✓

Candidate D - reversed limits

$\int_{1}^{-2}\left((x^3-2x^2-4x+1)-(x-5)\right)dx$ •1 ✓

$\frac{x^4}{4}-\frac{2x^3}{3}-\frac{5x^2}{2}+6x$ •3 ✓

$\left(\frac{(-2)^4}{4}-\frac{2(-2)^3}{3}-\frac{5(-2)^2}{2}+6(-2)\right)$

$-\left(\frac{(1)^4}{4}-\frac{2(1)^3}{3}-\frac{5(1)^2}{2}+6(1)\right)$ •4 ✓

$-\frac{63}{4}$

So Area $=\frac{63}{4}$ •2 ✓ •5 ✓

Candidate E - 'upper' – 'lower'

$=x^3-2x^2-5x+6$

$\int_{-2}^{1}\left(x^3-2x^2-5x+6\right)dx$ •1 ✓ •2 ✓

$\frac{x^4}{4}-\frac{2x^3}{3}-\frac{5x^2}{2}+6x$ •3 ✓

$\frac{37}{12}-\left(-\frac{38}{3}\right)$ •4 ✓

$\frac{63}{4}$ •5 ✓