

7.

$$\sin x + 2 = 3 \cos 2x$$

$$\sin x + 2 = 3(1 - 2\sin^2 x)$$

$$\sin x + 2 = 3 - 6\sin^2 x$$

$$6\sin^2 x + \sin x - 1 = 0$$

$$(3\sin x - 1)(2\sin x + 1) = 0$$

$$3\sin x - 1 = 0$$

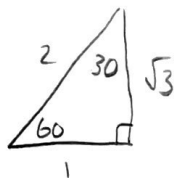
$$2\sin x + 1 = 0$$

$$\sin x = \frac{1}{3}$$

$$\sin x = -\frac{1}{2}$$

$$x = 19.5^\circ, 160.5^\circ$$

$$x = 210^\circ, 330^\circ$$



$$\sin^{-1}\left(\frac{1}{2}\right) = 30^\circ$$

S	A
T	C
✓	✓

Question			Generic scheme	Illustrative scheme	Max mark
7.			<ul style="list-style-type: none"> •¹ use double angle formula to express equation in terms of $\sin x^\circ$ •² arrange in standard quadratic form •³ factorise or use quadratic formula •⁴ solve for $\sin x^\circ$ •⁵ solve for x 	<ul style="list-style-type: none"> •¹ $\dots = 3(1 - 2\sin^2 x^\circ)$ •² $6\sin^2 x^\circ + \sin x^\circ - 1 = 0$ •³ $(3\sin x^\circ - 1)(2\sin x^\circ + 1) (= 0)$ or $\sin x^\circ = \frac{-1 \pm \sqrt{25}}{12}$ •⁴ $\sin x^\circ = \frac{1}{3},$ •⁵ $\sin x^\circ = -\frac{1}{2}$ •⁵ 19.47 ..., 160.52 ..., 210, 330 	5

Notes:

1. Substituting $1 - 2\sin^2 A$ or $1 - 2\sin^2 \alpha$ for $\cos 2x^\circ$ at the •¹ stage should be treated as bad form provided the equation is written in terms of x at •² stage. Otherwise, •¹ is not available.
2. Do not penalise the omission of degree signs.
3. ‘= 0’ must appear by •³ stage for •² to be awarded. However, for candidates using the quadratic formula to solve the equation, ‘= 0’ must appear at •² stage for •² to be awarded.
4. Candidates may express the equation obtained at •² in the form $6S^2 + S - 1 = 0$, $6x^2 + x - 1 = 0$ or using any other dummy variable at the •³ stage. In these cases, award •³ for $(3S - 1)(2S + 1)$ or $(3x - 1)(2x + 1)$.
However, •⁴ is only available if $\sin x^\circ$ appears explicitly at this stage - see Candidate A.
5. The equation $1 - 6\sin^2 x^\circ - \sin x^\circ = 0$ does not gain •² unless •³ has been awarded.
6. •³ is awarded for identifying the factors of the quadratic obtained at •² eg “ $3\sin x^\circ - 1 = 0$ and $2\sin x^\circ + 1 = 0$ ”.
7. •⁴ and •⁵ are only available as a consequence of trying to solve a quadratic equation - see Candidate B.
8. •³, •⁴ and •⁵ are not available for any attempt to solve a quadratic equation written in the form $ax^2 + bx = c$ - see Candidate C.
9. •⁵ is only available where at least one of the equations at •⁴ leads to two solutions for x .
10. Do not penalise additional (correct) solutions at •⁵. However see Candidates E and F.
11. Accept answers which round to 19, 19.5 and 161.

Question	Generic scheme	Illustrative scheme	Max mark
7. (continued)			
Commonly Observed Responses:			
Candidate A \vdots $6S^2 + S - 1 = 0$ $(3S - 1)(2S + 1) = 0$ $S = \frac{1}{3}, S = -\frac{1}{2}$ $x = 19.5, 160.5, 210, 330$	$\bullet^1 \checkmark \bullet^2 \checkmark$ $\bullet^3 \checkmark$ $\bullet^4 \wedge$ $\bullet^5 \boxed{\checkmark}_1$	Candidate B - not solving a quadratic \vdots $6 \sin^2 x^\circ + \sin x^\circ - 1 = 0$ $7 \sin x^\circ - 1 = 0$ $\sin x^\circ = \frac{1}{7}$ $x = 8.2$	$\bullet^1 \checkmark$ $\bullet^2 \checkmark$ $\bullet^3 \times$ $\bullet^4 \boxed{\checkmark}_2$ $\bullet^5 \boxed{\checkmark}_2$
Candidate C - not in standard quadratic form $\sin x^\circ + 2 = 3 - 6 \sin^2 x^\circ$ $6 \sin^2 x^\circ + \sin x^\circ = 1$ $\sin x^\circ (6 \sin x^\circ + 1) = 1$ $\sin x^\circ = 1 \quad 6 \sin x^\circ + 5 = 1$ $\Rightarrow \sin x = -\frac{4}{6}$ $90, 221.8, 318.2$	$\bullet^1 \checkmark$ $\bullet^2 \boxed{\checkmark}_2$ $\bullet^3 \boxed{\checkmark}_2$ $\bullet^4 \times$ $\bullet^5 \times$	Candidate D - reading $\cos 2x^\circ$ as $\cos^2 x^\circ$ $\sin x^\circ + 2 = 3 \cos^2 x^\circ$ $\sin x^\circ + 2 = 3(1 - \sin^2 x^\circ)$ $3 \sin^2 x^\circ + \sin x^\circ - 1 = 0$ $\sin x^\circ = \frac{-1 \pm \sqrt{13}}{6}$ $\sin x^\circ = 0.434..., \sin x^\circ = -0.767...$ $25.7, 154.3, 230.1, 309.9$	$\bullet^1 \times$ $\bullet^2 \boxed{\checkmark}_1$ $\bullet^3 \boxed{\checkmark}_1$ $\bullet^4 \boxed{\checkmark}_1$ $\bullet^5 \boxed{\checkmark}_1$
Candidate E \vdots $(3 \sin x^\circ - 1)(2 \sin x^\circ + 1) = 0$ $\sin x^\circ = \frac{1}{3}, \sin x^\circ = -\frac{1}{2}$ $x = 19, x = 161 \quad x = 30$ $x = 210, x = 330$ However, where the final solution(s) are clearly identified by the candidate award	$\bullet^1 \checkmark \bullet^2 \checkmark$ $\bullet^3 \checkmark$ $\bullet^4 \checkmark$ $\bullet^5 \times$	Candidate F \vdots $(3 \sin x^\circ - 1)(2 \sin x^\circ + 1) = 0$ $\sin x^\circ = \frac{1}{3}, \sin x^\circ = -\frac{1}{2}$ $x = 19, 161, 30, 210, 330$	$\bullet^1 \checkmark \bullet^2 \checkmark$ $\bullet^3 \checkmark$ $\bullet^4 \checkmark$ $\bullet^5 \times$