6.

$$f(f^{-1}(x)) = x$$

$$\frac{2}{f^{-1}(x)} + 3 = x$$

$$\frac{2}{f^{-1}(x)} = x - 3$$

$$f^{-1}(x) = \frac{2}{x-3}$$

Question	Generic scheme	Illustrative scheme	Max mark
6.	**Method 1**	**Method 1**	**3**
	•1 equate composite function to x	•1 $f\left(f^{-1}(x)\right)=x$	
	•2 write $f\left(f^{-1}(x)\right)$ in terms of $f^{-1}(x)$	•2 $x=\frac{2}{f^{-1}(x)}+3$	
	•3 state inverse function	•3 $f^{-1}(x)=\frac{2}{x-3}$	
	Method 2	**Method 2**	
	•1 write as $y=f(x)$ and start to rearrange	•1 $y=f(x)\Rightarrow x=f^{-1}(y)$ $y-3=\frac{2}{x}$	
	•2 express x in terms of y	•2 $x=\frac{2}{y-3}$	
	•3 state inverse function	•3 $f^{-1}(y)=\frac{2}{y-3}$ $\Rightarrow f^{-1}(x)=\frac{2}{x-3}$	

Notes:

1. In Method, 1 accept $x=\frac{2}{f^{-1}(x)}+3$ for •1 and •2.
2. In Method 2, accept '$y-3=\frac{2}{x}$' without reference to $y=f(x)\Rightarrow x=f^{-1}(y)$ at •1.
3. In Method 2, accept $f^{-1}(x)=\frac{2}{x-3}$ without reference to $f^{-1}(y)$ at •3.
4. In Method 2, beware of candidates with working where each line is not mathematically equivalent - see Candidates A and B for example.
5. At •3 stage, accept f^{-1} written in terms of any dummy variable eg $f^{-1}(y)=\frac{2}{y-3}$.
6. $y=\frac{2}{x-3}$ does not gain •3.
7. $f^{-1}(x)=\frac{2}{x-3}$ with no working gains 3/3.
8. In Method 2, where candidates make multiple algebraic errors at the •2 stage, •3 is still available.

Question	Generic scheme	Illustrative scheme	Max mark
6.	(continued)		

Commonly Observed Responses:

Candidate A

$f(x)=\frac{2}{x}+3$

$y=\frac{2}{x}+3$

$y-3=\frac{2}{x}$

$x=\frac{2}{y-3}$ $\bullet^1$ ✓ $\bullet^2$ ✓

$y=\frac{2}{x-3}$ $\bullet^3$ ×

$f^{-1}(x)=\frac{2}{x-3}$

Candidate B

$f(x)=\frac{2}{x}+3$

$y=\frac{2}{x}+3$

$x=\frac{2}{y}+3$ $\bullet^1$ ×

$x-3=\frac{2}{y}$

$y=\frac{2}{x-3}$ $\bullet^2$ ✓1

$f^{-1}(x)=\frac{2}{x-3}$ $\bullet^3$ ✓1

Candidate C - BEWARE

$f'(x)=\ldots$ $\bullet^3$ ×

Candidate D

$x\to\frac{1}{x}\to\frac{2}{x}\to\frac{2}{x}+3=f(x)$

$\times 2\to+3$

$\therefore -3\to\div 2$ $\bullet^1$ ✓

$\frac{2}{x-3}$ (invert) $\bullet^2$ ✓

$f^{-1}(x)=\frac{2}{x-3}$ $\bullet^3$ ✓

Candidate E

⋮ $\bullet^1$ ✓ $\bullet^2$ ✓

$f^{-1}(x)=\left(\frac{x-3}{2}\right)^{-1}$ $\bullet^3$ ✓

Candidate F

⋮ $\bullet^1$ ✓ $\bullet^2$ ✓

$f^{-1}(x)=\sqrt[-1]{\frac{x-3}{2}}$ $\bullet^3$ ✓

Candidate G

$y=\frac{2}{x}+3$

$xy=5$ $\bullet^1$ ×

$x=\frac{5}{y}$ $\bullet^2$ ✓2

$f^{-1}(x)=\frac{5}{x}$ $\bullet^3$ ✓1

However

$f^{-1}(x)=\frac{2+3}{x}$ $\bullet^3$ ^