QUESTION NUMBER

DO NOT WRITE IN THIS MARGIN

14.(a) (i)

$$A = 6x^2 + 2xh + 3xh + 2xh + 3xh$$

$$A = 6x^2 + 10xh$$

14.(a) (ii)

$$7200 = 6x^2 + 10xh$$

$$h = \frac{7200 - 6x^2}{10x}$$

$$V = (3x)(2x)\left(\frac{7200 - 6x^2}{10x}\right)$$

$$V = 6x^2\left(\frac{7200 - 6x^2}{10x}\right)$$

$$V = 4320x - \frac{18}{5}x^3 \text{ as req'd.}$$

QUESTION NUMBER

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14.(b)

$$V(x) = 4320x - \frac{18}{5}x^3$$

$$V'(x) = 4320 - \frac{54}{5}x^2$$

$$4320 - \frac{54}{5}x^2 = 0$$

$$\frac{54}{5}x^2 = 4320$$

$$x^2 = 400$$

$$x = \pm 20 \longrightarrow x = 20$$

	$\xrightarrow{10}$	20	$\xrightarrow{30}$
$V'(x)$	+	0	−
shape	↗	→	↘

Max when $x = 20$

<table>
<tr><th colspan="3">Question</th><th>Generic scheme</th><th>Illustrative scheme</th><th>Max mark</th></tr>
<tr><td>14.</td><td>(a)</td><td>(i)</td><td>•¹ express A in terms of x and h</td><td>•¹ $(A=)6x^2+10xh$</td><td>1</td></tr>
<tr><td></td><td></td><td>(ii)</td><td>•² express h in terms of x

•³ substitute for h and demonstrate result</td><td>•² $h=\dfrac{7200-6x^2}{10x}$

•³ $V=3x\times 2x\times\left(\dfrac{7200-6x^2}{10x}\right)$ leading to
$V=4320x-\dfrac{18}{5}x^3$</td><td>2</td></tr>
<tr><td colspan="6">Notes:</td></tr>
<tr><td colspan="6">1. Accept unsimplified expressions for •¹.
2. •² is only available where the (simplified) expression for A contains at least 2 terms.
3. The substitution for h at •³ must be clearly shown for •³ to be awarded.</td></tr>
<tr><td colspan="6">Commonly Observed Responses:</td></tr>
<tr><td colspan="6"></td></tr>
<tr><td></td><td>(b)</td><td></td><td>•⁴ differentiate

•⁵ equate expression for derivative to 0

•⁶ solve for x

•⁷ verify nature</td><td>•⁴ $4320-\dfrac{54}{5}x^2$

•⁵ $4320-\dfrac{54}{5}x^2=0$

•⁶ 20

•⁷ table of signs for a derivative
$$\begin{array}{r|c|c|c|}x & \dots & 20 & \dots \\ \hline V'(x) & + & 0 & - \\ \text{shape} & / & \text{—} & \backslash \end{array}$$
$\therefore$ maximum (when $x=20$)</td><td>4</td></tr>
<tr><td colspan="6">Notes:</td></tr>
<tr><td colspan="6">4. For any approach which does not use differentiation award 0/4.
5. •⁵ can be awarded for $\dfrac{54}{5}x^2=4320$.
6. For candidates who integrate any term at the •⁴ stage, only •⁵ is available on follow through for setting their ‘derivative’ to 0.
7. Ignore the appearance of −20 at mark •⁶.
8. Where −20 is considered in a nature table (or second derivative), “$x=20$” must be clearly identified as leading to the maximum area.
9. •⁶ and •⁷ are not available to candidates who state that the maximum exists at a negative value of x.
10. Do not penalise statements such as “max volume is 20” or “max is 20” at •⁷.</td></tr>
</table>

Question	Generic scheme	Illustrative scheme	Max mark
14. (continued)			

Commonly Observed Responses:

Candidate A - second derivative

$V''(x) = -\frac{108}{5}x$

$V''(20) < 0 \therefore$ maximum $\quad \bullet^7$ ✓

Candidate B - beware of multiplying before equating

$V'(x) = 4320 - \frac{54}{5}x^2 \quad \bullet^4$ ✓

$V'(x) = 21600 - 54x^2$

$21600 - 54x^2 = 0 \quad \bullet^5$ ✗

$x = 20 \quad \bullet^6$ $\boxed{\checkmark_1}$

Candidate C

Stationary points when $V'(x) = 0$

$V'(x) = 4320 - \frac{54}{5}x^2 \quad \bullet^4$ ✓ $\bullet^5$ ✓

For the table of signs for a derivative, accept:

x	20^-	20	20^+
$V'(x)$	$+$	0	$-$
Slope or shape	/	—	\

x	$\rightarrow$	20	$\rightarrow$
$V'(x)$	$+$	0	$-$
Slope or shape	/	—	\

Arrows are taken to mean 'in the neighbourhood of'

x	a	20	b
$V'(x)$	$+$	0	$-$
Slope or shape	/	—	\

Where $a < 20$ and $b > 20$

For the table of signs for a derivative, accept:

x	$\rightarrow$	-20	$\rightarrow$	20	$\rightarrow$
$V'(x)$	$-$	0	$+$	0	$-$
Slope or shape	\	—	/	—	\

Since the function is continuous $-20 \rightarrow 20$ is acceptable

x	a	-20	b	20	c
$V'(x)$	$-$	0	$+$	0	$-$
Slope or shape	\	—	/	—	\

Since the function is continuous $-20 < b < 20$ is acceptable

- For this question do not penalise the omission of 'x' or the word 'shape'/'slope'.
- Stating values of $V'(x)$ is an acceptable alternative to writing '+' or '–' signs.
- Acceptable variations of $V'(x)$ are: V', $\frac{dV}{dx}$, and $4320 - \frac{54}{5}x^2$. Accept $\frac{dy}{dx}$ only where candidates have previously used $y = 4320x - \frac{18}{5}x^3$ in their working.