QUESTION NUMBER

8.

DO NOT WRITE IN THIS MARGIN

$f(x) = x^3 + 3x^2 - 9x + 5$

$f'(x) = 3x^2 + 6x - 9$

$3x^2 + 6x - 9 = 0$

$x^2 + 2x - 3 = 0$

$(x+3)(x-1) = 0$

$x = -3 \qquad x = 1$

$y = 32 \qquad y = 0$

$(-3, 32) \qquad (1, 0)$

	$\xrightarrow{-4}$	-3	$\xrightarrow{-2}$
$f'(x)$	$+$	0	$-$
shape	↗	→	↘

Max @ $(-3, 32)$

	$\xrightarrow{0}$	1	$\xrightarrow{2}$
$f'(x)$	$-$	0	$+$
shape	↘	→	↗

Min @ $(1, 0)$

Question	Generic scheme	Illustrative scheme	Max mark
8.	$\bullet^1$ start to differentiate	$\bullet^1$ $3x^2\ldots$ or $\ldots+6x\ldots$ or $\ldots-9$	**6**
	$\bullet^2$ complete differentiation and equate to 0	$\bullet^2$ $3x^2+6x-9=0$	
	$\bullet^3$ solve for x	$\bullet^3$ -3 and 1	
	$\bullet^4$ process for y	$\bullet^4$ 32 and 0	
	$\bullet^5$ construct nature table(s)	$\bullet^5$ $\bullet^5$ $\bullet^6$ x | … | -3 | … | 1 | … $f'(x)$ | + | 0 | − | 0 | + shape | ↗ | — | ↘ | — | ↗	
	$\bullet^6$ interpret and state conclusions	$\bullet^6$ max at $(-3,32)$; min at $(1,0)$	

Notes:

1. For a numerical approach award 0/6.
2. $\bullet^2$ is only available if '$=0$' appears at the $\bullet^2$ stage or in working leading to $\bullet^3$, however see Candidates A and B.
3. Candidates who equate their derivative to 0, may use division by 3 as a strategy - see candidates B, C and D.
4. $\bullet^3$ is available to candidates who factorise **their** derivative from $\bullet^2$ as long as it is of equivalent difficulty.
5. $\bullet^3$ and $\bullet^4$ may be awarded vertically.
6. $\bullet^5$ is not available where any errors are made in calculating values of $f'(x)$.
7. $\bullet^5$ and $\bullet^6$ may be awarded vertically.
8. $\bullet^6$ is still available in cases where a candidates table of signs does not lead legitimately to a maximum/minimum shape.
9. Candidates may use the second derivative - see Candidates E and F.
10. Accept "max when $x=-3$" **and** "min when $x=1$" for $\bullet^6$.

Commonly Observed Responses:

Candidate A

Stationary points when $f'(x)=0$

$f'(x)=3x^2+6x-9$ $\bullet^1$ ✓ $\bullet^2$ ✓

$f'(x)=3(x+3)(x-1)$

$x=-3,\ 1$ $\bullet^3$ ✓

Candidate B

Stationary points when $f'(x)=0$

$f'(x)=3x^2+6x-9$ $\bullet^1$ ✓ $\bullet^2$ ✓

⋮

$f'(x)=(x+3)(x-1)$

$x=-3,\ 1$ $\bullet^3$ ✓

Candidate C - division by 3

$3x^2+6x-9=0$ $\bullet^1$ ✓ $\bullet^2$ ✓

$x^2+2x-3=0$

$x=-3,\ 1$ $\bullet^3$ ✓

Candidate D - derivative never equated to 0

$3x^2+6x-9$ $\bullet^1$ ✓ $\bullet^2$ ^

$x^2+2x-3=0$

$x=-3,\ 1$ $\bullet^3$ $\checkmark_1$

Question	Generic scheme	Illustrative scheme	Max mark

8.(continued)

Commonly Observed Responses:

Candidate E - second derivative

$f''(x)=6x+6$

$\bullet^5$ ✓ $\bullet^6$ ✓

$f''(-3)<0$ $f''(1)>0$ $\bullet^5$ ✓

so max at $(-3,32)$ so min at $(1,0)$ $\bullet^6$ ✓

Candidate F - second derivative

$f''(x)=6x+6$

$\bullet^5$ ✓ $\bullet^6$ ✓

$f''(-3)=-12$, $f''(1)=12$ $\bullet^5$ ✓

$-12<0$ $12>0$

so max at $(-3,32)$ so min at $(1,0)$ $\bullet^6$ ✓

For the table of signs for a derivative, accept:

x	-3^-	-3	-3^+
$f'(x)$	+	0	−
Slope or shape	/	———	\

AND

x	1^-	1	1^+
$f'(x)$	−	0	+
Slope or shape	\	———	/

x	$\to$	-3	$\to$
$f'(x)$	+	0	−
Slope or shape	/	———	\

Arrows are taken to mean 'in the neighbourhood of'

AND

x	$\to$	1	$\to$
$f'(x)$	−	0	+
Slope or shape	\	———	/

Arrows are taken to mean 'in the neighbourhood of'

x	a	-3	b
$f'(x)$	+	0	−
Slope or shape	/	———	\

Where $a<-3$ and $-3<b<1$

AND

x	c	1	d
$f'(x)$	−	0	+
Slope or shape	\	———	/

Where $-3<c<1$ and $d>1$

For the table of signs for a derivative, accept:

x	$\to$	-3	$\to$	1	$\to$
$f'(x)$	+	0	−	0	+
Slope or shape	/	———	\	———	/

Since the function is continuous $-3\to 1$ is acceptable

x	a	-3	b	1	c
$f'(x)$	+	0	−	0	+
Slope or shape	/	———	\	———	/

Since the function is continuous $-3<b<1$ is acceptable

- For this question do not penalise the omission of 'x' or the word 'shape'/'slope'.
- Stating values of $f'(x)$ is an acceptable alternative to writing '+' or '−' signs.
- Acceptable variations of $f'(x)$ are: f', $\frac{df}{dx}$, $\frac{dy}{dx}$, $3x^2+6x-9$ and $3(x+3)(x-1)$

  but NOT x^2+2x-3 or $(x+3)(x-1)$.