8. 
$$f(x) = x^{3} + 3x^{2} - 9x + 5$$

$$f'(x) = 3x^{2} + 6x - 9$$

$$3x^{2} + 6x - 9 = 0$$

$$5x^{2} + 2x - 3 = 0$$

$$(5x + 3)(x - 1) = 0$$

$$x = -3$$
  $x = 1$   
 $y = 32$   $y = 0$   
 $(-3,32)$   $(1,0)$ 

<del>-4</del> -3 <del>-</del> 7	
f(x) + 0	f(x) - 0 +
$f(x) + 0 -$ Shape $1 \rightarrow y$	shape \
Max@ (-3,32)	Min @ (1,0)

Qı	uestio	n	Generic scheme	Illustrative scheme	
8.			•¹ start to differentiate	• $3x^2$ or + 6x or 9	6
			•² complete differentiation and equate to 0		
			$\bullet$ <sup>3</sup> solve for $x$	•³ •³ -3 and 1	
			• <sup>4</sup> process for <i>y</i>	• <sup>4</sup> 32 and 0	
			•5 construct nature table(s)	•5 •6	
				x3 1	
				$\begin{array}{ c c c c c c c c c c c c c c c c c c c$	
			• 6 interpret and state conclusions	•6 max at $(-3,32)$ ; min at $(1,0)$	

#### Notes:

- 1. For a numerical approach award 0/6.
- 2.  $\bullet^2$  is only available if '= 0' appears at the  $\bullet^2$  stage or in working leading to  $\bullet^3$ , however see Candidates A and B.
- 3. Candidates who equate their derivative to 0, may use division by 3 as a strategy see candidates B, C and D.
- 4. •³ is available to candidates who factorise **their** derivative from •² as long as it is of equivalent difficulty.
- 5.  $\bullet^3$  and  $\bullet^4$  may be awarded vertically.
- 6. 5 is not available where any errors are made in calculating values of f'(x).
- 7.  $\bullet^5$  and  $\bullet^6$  may be awarded vertically.
- 8. 6 is still available in cases where a candidates table of signs does not lead legitimately to a maximum/minimum shape.
- 9. Candidates may use the second derivative see Candidates E and F.
- 10. Accept "max when x = -3" and "min when x = 1" for  $\bullet$ 6.

#### Commonly Observed Responses:

Commonly Cases for the periods					
Candidate A		Candidate B			
Stationary points when $f$	$\mathbf{C}'(x) = 0$	Stationary points when	Stationary points when $f'(x) = 0$		
$f'(x) = 3x^2 + 6x - 9$	•¹ ✓ •² ✓	$f'(x) = 3x^2 + 6x - 9$	•¹ <b>✓</b> •² <b>✓</b>		
f'(x) = 3(x+3)(x-1)		:			
x = -3, 1	•³ ✓	f'(x) = (x+3)(x-1) $x = -3, 1$	•³ <b>✓</b>		
Candidate C - division by	, 3	Candidate D - derivativ	•		
_		_	-		
$3x^2 + 6x - 9 = 0$	•¹ <b>✓</b> •² <b>✓</b>	$3x^2 + 6x - 9$	•¹ ✓ •² ∧		
$x^2 + 2x - 3 = 0$		$x^2 + 2x - 3 = 0$	. —		
x = -3, 1	•³ <b>✓</b>	x = -3, 1	• <sup>3</sup> 🗸 1		

#### 8.(continued)

## **Commonly Observed Responses:**

### Candidate E - second derivative

$$f''(x) = 6x + 6$$

•<sup>5</sup> ✓

$$f''(-3) < 0$$

Slope or shape

so max at (-3,32)

f''(1) > 0

#### Candidate F - second derivative

f''(x) = 6x + 6

$$f''(-3) = -12$$
,

f''(1) = 12

so min at (1,0) so max at (-3,32)

For the table of signs for a derivative, accept:

x	<b>-3</b> <sup>-</sup>	-3	$-3^{+}$
f'(x)	+	0	_
Slope or	/		\
or			
shape			

AND

# f'(x)

Slope shape

> Arrows are taken to mean 'in the neighbourhood of'

#### AND

<u>x</u>	$\rightarrow$	1	$\rightarrow$
f'(x)	-	0	+
Slope or	\		/
or			
shape			_ /

Arrows are taken to mean 'in the neighbourhood of'

## f'(x)Slope or shape

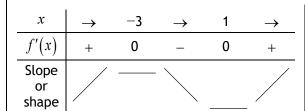
Where a < -3 and -3 < b < 1

#### **AND**

$\boldsymbol{x}$	c	1	d
f'(x)	-	0	+
Slope or	\		/
or			
shape			_ /

Where -3 < c < 1 and d > 1

For the table of signs for a derivative, accept:



Since the function is continuous  $-3 \rightarrow 1$  is acceptable

x	а	-3	b	1	С
f'(x)	+	0	_	0	+
Slope or shape					

Since the function is continuous -3 < b < 1 is acceptable

- For this question do not penalise the omission of 'x' or the word 'shape'/'slope'.
- Stating values of f'(x) is an acceptable alternative to writing '+' or '-' signs.
- Acceptable variations of f'(x) are: f',  $\frac{df}{dx}$ ,  $\frac{dy}{dx}$ ,  $3x^2 + 6x 9$  and 3(x+3)(x-1)but NOT  $x^2 + 2x - 3$  or (x+3)(x-1).