

10.(a)

$$\begin{array}{r|rrrrr}
 -5 & x^4 & x^3 & x^2 & x & r \\
 & 1 & 3 & -7 & 9 & -30 \\
 & \downarrow & & & & \\
 & & -5 & 10 & -15 & 30 \\
 \hline
 & 1 & -2 & 3 & -6 & 0
 \end{array}$$

Since $r=0$,
 $(x+5)$ is
 a factor

10.(b)

$$(x+5)(x^3 - 2x^2 + 3x - 6) = 0$$

$$\begin{array}{r|rrrr}
 2 & x^3 & x^2 & x & r \\
 & 1 & -2 & 3 & -6 \\
 & \downarrow & & & \\
 & & 2 & 0 & 6 \\
 \hline
 & 1 & 0 & 3 & 0
 \end{array}$$

Since $r=0$, $(x+2)$ is a factor

$$(x+5)(x-2)(x^2+3)=0$$

$$x = -5 \quad x = 2 \quad x^2 + 3 = 0$$

has no real
 solutions

$$(b^2 - 4ac = -12)$$

Question			Generic scheme	Illustrative scheme	Max mark
10.	(a)		<p>•¹ use -5 in synthetic division or evaluation of quartic</p> <p>•² complete division/evaluation and interpret result</p>	<p>•¹</p> $\begin{array}{r rrrrr} -5 & 1 & 3 & -7 & 9 & -30 \\ & & 1 & & & \end{array}$ <p>or</p> $(-5)^4 + 3 \times (-5)^3 - 7 \times (-5)^2 + 9 \times (-5) - 30$ <p>•²</p> $\begin{array}{r rrrrr} -5 & 1 & 3 & -7 & 9 & -30 \\ & & -5 & 10 & -15 & 30 \\ \hline & 1 & -2 & 3 & -6 & 0 \end{array}$ <p>Remainder = 0 $\therefore (x+5)$ is a factor OR $f(-5) = 0 \therefore (x+5)$ is a factor</p>	2

Notes:

- Communication at •² must be consistent with working at that stage i.e. a candidate's working must arrive legitimately at 0 before •² can be awarded.
- Accept any of the following for •²:
 - ' $f(-5) = 0$ so $(x+5)$ is a factor'
 - 'since remainder = 0, it is a factor'
 - the '0' from any method linked to the word 'factor' by 'so', 'hence', \therefore , \rightarrow , \Rightarrow etc.
- Do not accept any of the following for •²:
 - double underlining the '0' or boxing the '0' without comment
 - ' $x = -5$ is a factor', '... is a root'
 - the word 'factor' only, with no link.

Commonly Observed Responses:

Candidate A - grid method

$$\begin{array}{r|rrrr} x^3 & & & & \\ x & x^4 & -2x^3 & & \\ 5 & 5x^3 & & & \end{array} \quad \bullet^1 \checkmark$$

$$\begin{array}{r|rrrr} x^3 & -2x^2 & +3x & -6 \\ x & x^4 & -2x^3 & +3x^2 & -6x \\ 5 & 5x^3 & -10x^2 & +15x & -30 \end{array}$$

with no remainder

$\therefore (x+5)$ is a factor •² ✓

Candidate B - grid method

$$\begin{array}{r|rrrr} x^3 & & & & \\ x & x^4 & -2x^3 & & \\ 5 & 5x^3 & & & \end{array} \quad \bullet^1 \checkmark$$

$$\begin{array}{r|rrrr} x^3 & -2x^2 & +3x & -6 \\ x & x^4 & -2x^3 & +3x^2 & -6x \\ 5 & 5x^3 & -10x^2 & +15x & -30 \end{array}$$

$\therefore (x+5)(x^3 - 2x^2 + 3x - 6) = x^4 + 3x^3 - 7x^2 + 9x - 30$

$\therefore (x+5)$ is a factor •² ✓

Question			Generic scheme	Illustrative scheme	Max mark
10.	(b)		<p>•³ identify cubic and attempt to factorise</p> <p>•⁴ find second factor</p> <p>•⁵ identify quadratic</p> <p>•⁶ interpret lack of solutions of quadratic</p> <p>•⁷ state solutions</p>	<p>•³ eg</p> $\begin{array}{r rrrr} 1 & 1 & -2 & 3 & -6 \\ & & 1 & -1 & \\ \hline & 1 & -1 & & \end{array}$ <p>or</p> $\begin{array}{r rrrr} 2 & 1 & -2 & 3 & -6 \\ & & 2 & 0 & \\ \hline & 1 & 0 & & \end{array}$ <p>•⁴</p> $\begin{array}{r rrrr} 2 & 1 & -2 & 3 & -6 \\ & & 2 & 0 & 6 \\ \hline & 1 & 0 & 3 & 0 \end{array}$ <p>leading to $(x-2)$ or $x=2$</p> <p>•⁵ $x^2 + 3$</p> <p>•⁶ $b^2 - 4ac = -12 < 0$ \therefore no (further real) solutions OR $x^2 = -3$ or $x^2 = -3$ \therefore no (further real) solutions</p> <p>•⁷ $x = -5, x = 2$</p>	5

Notes:

4. Candidates who arrive at $(x+5)(x-2)(x^2+3)$ by using algebraic long division or by inspection gain •³, •⁴ and •⁵.
5. Evidence for •⁶ may appear in the quadratic formula.
6. At •⁶ accept interpretations such as “no further roots”, “no solutions” and “cannot factorise further” with justification.
7. At •⁶ accept $x = \sqrt{-3}$ leading to “not possible” and “not real”.
8. Where there is no reference to $b^2 - 4ac$ accept ‘ $-12 < 0$ so no real roots’ with the remaining roots stated for •⁶ - see candidates E and F.
9. Do not accept any of the following for •⁶:
 - $(x+5)(x-2)(x^2+3)$ no further roots/cannot factorise further.
 - $(x+5)(x-2)(\dots \dots)(\dots \dots)$ no further roots/cannot factorise further.
10. Where the quadratic factor obtained at •⁵ can be factorised, •⁶ and •⁷ are not available.
11. •⁷ is only available where •⁶ has been awarded.

Question	Generic scheme	Illustrative scheme	Max mark																																							
10.(continued)																																										
Commonly Observed Responses:																																										
Candidate C $(x+5)(x-2)(x^2+3)$ $b^2-4ac=0-12<0$ so no solutions $x=-5, x=2$	$\bullet^5 \checkmark$ $\bullet^6 \checkmark$ $\bullet^7 \checkmark$	Candidate D $(x+5)(x-2)(x^2+3)$ $b^2-4ac<0$ so no solutions $x=-5, x=2$	$\bullet^5 \checkmark$ $\bullet^6 \wedge$ $\bullet^7 \boxed{\checkmark 2}$																																							
Candidate E $(x+5)(x-2)(x^2+3)$ $-12<0$ so no solutions $x=-5, x=2$	$\bullet^5 \checkmark$ $\bullet^6 \checkmark$ $\bullet^7 \checkmark$	Candidate F $(x+5)(x-2)(x^2+3)$ $-12<0$ so no solutions	$\bullet^5 \checkmark$ $\bullet^6 \wedge \bullet^7 \wedge$																																							
Candidate G - grid method (a) <table><tr><td></td><td>x^3</td><td>$-2x^2$</td><td>$3x$</td><td>-6</td></tr><tr><td>x</td><td>x^4</td><td>$-2x^3$</td><td>$+3x^2$</td><td>$-6x$</td></tr><tr><td>5</td><td>$5x^3$</td><td>$-10x^2$</td><td>$+15x$</td><td>-30</td></tr></table> (b) <table><tr><td></td><td>x^2</td><td>...</td><td>...</td></tr><tr><td>x</td><td>x^3</td><td>...</td><td>...</td></tr><tr><td>...</td><td>...</td><td>...</td><td>...</td></tr></table> $\bullet^3 \checkmark$ \bullet^3 is awarded for evidence of the cubic expression (which may be in the grid from part (a)) AND the terms in the diagonal boxes summing to the second and third terms in the cubic respectively. <table><tr><td></td><td>x^2</td><td>$0x$</td><td>3</td></tr><tr><td>x</td><td>x^3</td><td>0</td><td>$3x$</td></tr><tr><td>-2</td><td>$-2x^2$</td><td>0</td><td>-6</td></tr></table> $(x+5)(x-2)(x^2+3)$ $b^2-4ac=-12<0$ \therefore no more solutions $x=-5, x=2$			x^3	$-2x^2$	$3x$	-6	x	x^4	$-2x^3$	$+3x^2$	$-6x$	5	$5x^3$	$-10x^2$	$+15x$	-30		x^2	x	x^3		x^2	$0x$	3	x	x^3	0	$3x$	-2	$-2x^2$	0	-6	$\bullet^4 \checkmark$ $\bullet^5 \checkmark$ $\bullet^6 \checkmark$ $\bullet^7 \checkmark$	
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