8.a)
$$xy = 150$$
 $A = (x-3)(y-2)$
 $y = \frac{150}{x}$ $A = xy - 2x - 3y + 6$
 $= x(\frac{150}{2} - 2x - 3(\frac{150}{2}) + 6$
 $= 150 - 2x - 450 + 6$
 $= 156 - 2x - 450$ as required

Question		n	Generic scheme	Illustrative scheme	Max mark
8.	(a)		•¹ determine expression for area of pond	•¹ $(x-3)(y-2)$ stated or implied by •³	3
			• obtain expression for y	$\bullet^2 y = \frac{150}{x}$	
			•³ demonstrate result	$\bullet^3 A(x) = (x-3)\left(\frac{150}{x} - 2\right)$	
				$eg \ A(x) = \frac{150x}{x} - \frac{450}{x} - 2x + 6$ $A(x) = 156 - 2x - \frac{450}{x}$	
				$A(x) = 156 - 2x - \frac{430}{x}$	

Notes:

- 1. Accept any legitimate variations for the area of the pond in \bullet^1 , eg A = 150 2(x 3) 2(y)(1.5).
- 2. Do not penalise the omission of brackets at $ullet^1$. See Candidate A.
- 3. The substitution for y at \bullet^3 must be clearly shown for \bullet^3 to be available.

Commonly Observed Responses:

Candidate A

$$A(x) = x - 3 \times y - 2$$

$$A(x) = x - 3 \times \frac{150}{x} - 2$$

$$A(x) = x - 3 \times \frac{150}{x} - 2$$
$$A(x) = 156 - 2x - \frac{450}{x}$$

Question		n	Generic scheme	Illustrative scheme	Max mark
8.	(b)		$ullet^4$ express A in differentiable form	•4 $156 - 2x - 450x^{-1}$ stated or implied by •5	6
			• ⁵ differentiate	$\bullet^5 -2 + 450x^{-2}$	
			• equate expression for derivative to 0	$\bullet^6 -2 + 450x^{-2} = 0$	
			\bullet^7 solve for x	$\bullet^7 x = 15$	
			• ⁸ verify nature of stationary point	• ⁸ table of signs for derivative ∴ maximum or $A''(x) = -900x^{-3}$ and $A''(15) < 0$ ∴ maximum	
			•9 determine maximum area	$\bullet^9 A = 96 (m^2)$	

Notes:

- 4. For a numerical approach award 0/6.
- 5. 6 can be awarded for $450x^{-2} = 2$.
- 6. For candidates who integrate any term at the \bullet^5 stage, only \bullet^6 is available on follow through for setting their 'derivative' to 0.
- 7. \bullet^7 , \bullet^8 , and \bullet^9 are only available for working with a derivative which contains an index ≤ -2 .
- 8. $\sqrt{\frac{450}{2}}$ must be simplified at \bullet^7 or \bullet^8 for \bullet^7 to be awarded.
- 9. Ignore the appearance of -15 at mark \bullet^7 .
- 10. •8 is not available to candidates who consider a value of $x \le 0$ in the neighbourhood of 15.
- 11. 9 is still available in cases where a candidate's table of signs does not lead legitimately to a maximum at 8.
- 12. •8 and •9 are not available to candidates who state that the maximum exists at a negative value of x.

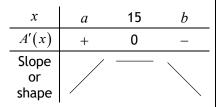
Question		n	Generic scheme	Illustrative scheme	Max mark
8.	(b)	(continued)			

Notes (continued)

For the table of signs for a derivative, accept:

x	15 ⁻	15	15 ⁺
A'(x)	+	0	_
Slope or shape	/		
or			
shape			

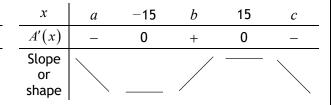
x	\rightarrow	15	\rightarrow
A'(x)	+	0	_
Slope or	/		\
or			
shape			



Arrow are taken to mean 'in the neighbourhood of'

Where 0 < a < 15 and b > 15

For the table of signs for a derivative, do not accept:



Since the function is discontinuous $-15 \rightarrow 15$ is not acceptable

Since the function is discontinuous -15 < b < 15 is not acceptable

- For this question do not penalise the omission of 'x' or the word 'shape'/'slope'.
- Stating values of A'(x) is an acceptable alternative to writing '+' or '-' signs.
- Acceptable variations of A'(x) are: A', a'(x), $\frac{dA}{dx}$, and $-2+450x^{-2}$.

Commonly Observed Responses:

Candidate B - differentiating over multiple lines

$$A'(x) = -2 - 450x^{-1}$$
$$A'(x) = -2 + 450x^{-2}$$

$$-2 + 450x^{-2} = 0$$

Candidate C - differentiating over multiple lines

$$A(x) = 156 - 2x - 450x^{-1}$$

$$A'(x) = -2 - 450x^{-1}$$

$$A'(x) = -2 + 450x^{-2}$$

$$-2 + 450x^{-2} = 0$$