8. a) $xy = 150$

$y = \frac{150}{x}$

$A = (x-3)(y-2)$

$A = xy - 2x - 3y + 6$

$= x\left(\frac{150}{x}\right) - 2x - 3\left(\frac{150}{x}\right) + 6$

$= 150 - 2x - \frac{450}{x} + 6$

$= 156 - 2x - \frac{450}{x}$ as required

Question			Generic scheme	Illustrative scheme	Max mark
8.	(a)		$\bullet^1$ determine expression for area of pond $\bullet^2$ obtain expression for y $\bullet^3$ demonstrate result	$\bullet^1$ $(x-3)(y-2)$ stated or implied by $\bullet^3$ $\bullet^2$ $y=\frac{150}{x}$ $\bullet^3$ $A(x)=(x-3)\left(\frac{150}{x}-2\right)$ eg $A(x)=\frac{150x}{x}-\frac{450}{x}-2x+6$ $A(x)=156-2x-\frac{450}{x}$	**3**

Notes:

1. Accept any legitimate variations for the area of the pond in $\bullet^1$, eg $A=150-2(x-3)-2(y)(1.5)$.
2. Do not penalise the omission of brackets at $\bullet^1$. See Candidate A.
3. The substitution for y at $\bullet^3$ must be clearly shown for $\bullet^3$ to be available.

Commonly Observed Responses:

Candidate A

$A(x)=x-3\times y-2$ $\bullet^1$ ✓

$A(x)=x-3\times\frac{150}{x}-2$ $\bullet^2$ ✓

$A(x)=156-2x-\frac{450}{x}$ $\bullet^3$ ^

Question			Generic scheme	Illustrative scheme	Max mark
8.	(b)		$\bullet^4$ express A in differentiable form	$\bullet^4$ $156-2x-450x^{-1}$ stated or implied by $\bullet^5$	**6**
			$\bullet^5$ differentiate	$\bullet^5$ $-2+450x^{-2}$	
			$\bullet^6$ equate expression for derivative to 0	$\bullet^6$ $-2+450x^{-2}=0$	
			$\bullet^7$ solve for x	$\bullet^7$ $x=15$	
			$\bullet^8$ verify nature of stationary point	$\bullet^8$ table of signs for derivative $\therefore$ maximum or $A''(x)=-900x^{-3}$ and $A''(15)<0$ $\therefore$ maximum	
			$\bullet^9$ determine maximum area	$\bullet^9$ $A=96\left(\text{m}^2\right)$	

Notes:

4. For a numerical approach award 0/6.
5. $\bullet^6$ can be awarded for $450x^{-2}=2$.
6. For candidates who integrate any term at the $\bullet^5$ stage, only $\bullet^6$ is available on follow through for setting their 'derivative' to 0.
7. $\bullet^7$, $\bullet^8$, and $\bullet^9$ are only available for working with a derivative which contains an index ≤ -2.
8. $\sqrt{\frac{450}{2}}$ must be simplified at $\bullet^7$ or $\bullet^8$ for $\bullet^7$ to be awarded.
9. Ignore the appearance of -15 at mark $\bullet^7$.
10. $\bullet^8$ is not available to candidates who consider a value of $x\leq 0$ in the neighbourhood of 15.
11. $\bullet^9$ is still available in cases where a candidate's table of signs does not lead legitimately to a maximum at $\bullet^8$.
12. $\bullet^8$ and $\bullet^9$ are not available to candidates who state that the maximum exists at a negative value of x.

Question	Generic scheme	Illustrative scheme	Max mark
8. (b)	**(continued)**		

Notes (continued)

For the table of signs for a derivative, accept:

x	15^-	15	15^+
$A'(x)$	+	0	−
Slope or shape	⟋	—	⟍

x	$\rightarrow$	15	$\rightarrow$
$A'(x)$	+	0	−
Slope or shape	⟋	—	⟍

Arrow are taken to mean 'in the neighbourhood of'

x	a	15	b
$A'(x)$	+	0	−
Slope or shape	⟋	—	⟍

Where $0 < a < 15$ and $b > 15$

For the table of signs for a derivative, **do not** accept:

x	$\rightarrow$	-15	$\rightarrow$	15	$\rightarrow$
$A'(x)$	−	0	+	0	−
Slope or shape	⟍	—	⟋	—	⟍

Since the function is discontinuous $-15 \rightarrow 15$ is not acceptable

x	a	-15	b	15	c
$A'(x)$	−	0	+	0	−
Slope or shape	⟍	—	⟋	—	⟍

Since the function is discontinuous $-15 < b < 15$ is not acceptable

- For this question do not penalise the omission of 'x' or the word 'shape'/'slope'.
- Stating values of $A'(x)$ is an acceptable alternative to writing '+' or '−' signs.
- Acceptable variations of $A'(x)$ are: A', $a'(x)$, $\frac{dA}{dx}$, and $-2+450x^{-2}$.

Commonly Observed Responses:

Candidate B - differentiating over multiple lines

$A'(x) = -2 - 450x^{-1}$ •4 ^

$A'(x) = -2 + 450x^{-2}$ •5 ✗

$-2 + 450x^{-2} = 0$ •6 ✓ 1

Candidate C - differentiating over multiple lines

$A(x) = 156 - 2x - 450x^{-1}$ •4 ✓

$A'(x) = -2 - 450x^{-1}$

$A'(x) = -2 + 450x^{-2}$ •5 ✗

$-2 + 450x^{-2} = 0$ •6 ✓ 1