4. a) $\int_{-1}^{2} x^3 - 5x^2 + 2x + 8 \, dx = \left[\frac{x^4}{4} - \frac{5x^3}{3} + \frac{2x^2}{2} + 8x\right]_{-1}^{2}$

$= \left(\frac{2^4}{4} - \frac{5(2)^3}{3} + 2^2 + 8(2)\right) - \left(\frac{(-1)^4}{4} - \frac{5(-1)^3}{3} + (-1)^2 + 8(-1)\right)$

$= \frac{32}{3} - \frac{-61}{12}$

$= \frac{63}{4}$ units^2

b) $\int_{2}^{4} \text{---} \ldots \, dx$

$= \left(\frac{4^4}{4} - \frac{5(4)^3}{3} + 4^2 + 8(4)\right) - \frac{32}{3}$

$= \frac{16}{3} - \frac{32}{3} = \frac{-16}{3}$

Total area $= \frac{63}{4} + \frac{16}{3} = \frac{253}{12}$ units^2

Question			Generic scheme	Illustrative scheme	Max mark
4.	(a)		•¹ state appropriate integral	•¹ $\int_{-1}^{2}\left(x^3-5x^2+2x+8\right)dx$	4
			•² integrate	•² $\frac{1}{4}x^4-\frac{5}{3}x^3+\frac{2x^2}{2}+8x$	
			•³ substitute limits	•³ $\left(\frac{1}{4}(2)^4-\frac{5}{3}(2)^3+(2)^2+8(2)\right)$ $-\left(\frac{1}{4}(-1)^4-\frac{5}{3}(-1)^3+(-1)^2+8(-1)\right)$	
			•⁴ evaluate area	•⁴ $\frac{63}{4}$ or 15.75	

Notes:

1. Limits and 'dx' must appear at the •¹ stage for •¹ to be awarded.
2. Where a candidate differentiates one or more terms at •², then •³ and •⁴ are not available.
3. Candidates who substitute limits without integrating, do not gain •³ or •⁴.
4. Do not penalise the inclusion of '$+c$'.
5. Do not penalise the continued appearance of the integral sign after •¹.
6. •⁴ is not available where solutions include statements such as $-\frac{63}{4}=\frac{63}{4}$. See Candidate C.

Commonly Observed Responses:

Candidate A		Candidate B - evidence of substitution using a calculator	
$\int_{-1}^{2}\left(x^3-5x^2+2x+8\right)$	•¹ ✗	$\int\left(x^3-5x^2+2x+8\right)dx$	•¹ ✗
$=\frac{1}{4}x^4-\frac{5}{3}x^3+\frac{2x^2}{2}+8x$	•² ✓	$=\frac{1}{4}x^4-\frac{5}{3}x^3+\frac{2x^2}{2}+8x$	•² ✓
	•³ ^	$=\frac{32}{3}-\left(-\frac{61}{12}\right)$	•³ ✓
$=\frac{63}{4}$	•⁴ ✓ 1	$=\frac{63}{4}$	•⁴ ✓

Candidate C - communication for •⁴	
$\int_{2}^{-1}\left(x^3-5x^2+2x+8\right)dx$	•¹ ✓
…	•² ✓ •³ ✓
$=-\frac{63}{4}$, hence area is $\frac{63}{4}$.	•⁴ ✓
However $-\frac{63}{4}=\frac{63}{4}$ square units does not gain •⁴	

Question			Generic scheme	Illustrative scheme	Max mark
4.	(b)		**Method 1**	**Method 1**	3
			•5 state appropriate integral	•5 $\int_2^4 \left(x^3-5x^2+2x+8\right)dx$	
			•6 evaluate integral	•6 $-\frac{16}{3}$	
			•7 interpret result and evaluate total area	•7 $\frac{253}{12}$ or 21.083…	
			Method 2	**Method 2**	
			•5 state appropriate integral	•5 $\int_2^4 \left(0-\left(x^3-5x^2+2x+8\right)\right)dx$	
			•6 substitute limits	•6 $-\left(\frac{1}{4}(4)^4-\frac{5}{3}(4)^3+(4)^2+8(4)\right)-\left(-\left(\frac{1}{4}(2)^4-\frac{5}{3}(2)^3+(2)^2+8(2)\right)\right)$	
			•7 evaluate total area	•7 $\frac{253}{12}$ or 21.083…	

Notes:

7. For candidates who only consider $\int_{-1}^{4}\dots dx$ or any other invalid integral, award 0/3.
8. In part (b), at •5 do not penalise the omission of 'dx'.
9. In Method 1, •5 may be awarded for $\left[\frac{1}{4}x^4-\frac{5}{3}x^3+\frac{2x^2}{2}+8x\right]_2^4$

   or $\left(\frac{1}{4}(4)^4-\frac{5}{3}(4)^3+(4)^2+8(4)\right)-\left(\frac{1}{4}(2)^4-\frac{5}{3}(2)^3+(2)^2+8(2)\right)$.
10. In Method 2, •5 may be awarded for $\left[\frac{1}{4}x^4-\frac{5}{3}x^3+\frac{2x^2}{2}+8x\right]_4^2$ or •5 and •6 may be awarded for

    $\left(\frac{1}{4}(2)^4-\frac{5}{3}(2)^3+(2)^2+8(2)\right)-\left(\frac{1}{4}(4)^4-\frac{5}{3}(4)^3+(4)^2+8(4)\right)$.
11. •7 is not available to candidates where solutions include statements such as $-\frac{16}{3}=\frac{16}{3}$ square units. See Candidate D.
12. In Method 1, where a candidate's integral leads to a positive value, •7 is not available.
13. Where a candidate has differentiated in both parts of the question see Candidate E.

Question			Generic scheme	Illustrative scheme	Max mark
4.	(b)	**(continued)**			

Commonly Observed Responses:

Candidate D - communication for •7

$\int_2^4 (x^3 - 5x^2 + 2x + 8)\,dx = -\frac{16}{3}$ •5 ✓ •6 ✓

$\frac{63}{4} + \frac{16}{3} = \frac{253}{12}$ •7 ✓

However, •7 is not available where statements such as "$-\frac{16}{3} = \frac{16}{3}$ square units" or "ignore negative" appear.

Candidate E - differentiation in (a) and (b)

(a) $\int_{-1}^{2} (x^3 - 5x^2 + 2x + 8)\,dx$ •1 ✓

$= 3x^2 - 10x + 2$ •2 ✗

$= \left(3(2)^2 - 10(2) + 2\right) - \left(3(-1)^2 - 10(-1) + 2\right)$ •3 ✗

$= -21$

Area = 21 •4 ✗

(b) $\left(3(4)^2 - 10(4) + 2\right) - \left(3(2)^2 - 10(2) + 2\right) = 16$ •5 ✓ •6 ✓ 1

Total Area $= 5$ •7 ✓ 2 see note 12