3a) $k\sin(x+a) = k\sin x\cos a + k\cos x\sin a$

$4\sin x + 5\cos x$

$k\cos a = 4$

$k\sin a = 5$

$\tan a = \frac{5}{4}$

$a = 51$

$k = \sqrt{4^2+5^2}$

$= \sqrt{41}$

$\sqrt{41}\sin(x+51)$

$\frac{51\times\pi}{180} = 0.89$

$\sqrt{41}\sin(x+0.89)$

b) $\sqrt{41}\sin(x+51) = 5.5$

$\sin(x+51) = 0.858\ldots$

$x+51 = 59°, 121, 419$

$x = 8°, 70°, 3\cancel{88}$

$x = 0.14, 1.22$

180
S | A
T | C

$180-59=$

$\frac{8\times\pi}{180} =$

$\frac{70\times\pi}{180} =$

Question			Generic scheme	Illustrative scheme	Max mark
3.	(a)		$\bullet^1$ use compound angle formula	$\bullet^1$ $k\sin x\cos a + k\cos x\sin a$ stated explicitly	4
			$\bullet^2$ compare coefficients	$\bullet^2$ $k\cos a = 4$ and $k\sin a = 5$ stated explicitly	
			$\bullet^3$ process for k	$\bullet^3$ $k = \sqrt{41}$	
			$\bullet^4$ process for a and express in required form	$\bullet^4$ $\sqrt{41}\sin(x + 0.896\ldots)$	

Notes:

1. Accept $k(\sin x\cos a + \cos x\sin a)$ at $\bullet^1$.
2. Treat $k\sin x\cos a + \cos x\sin a$ as bad form only if the equations at the $\bullet^2$ stage both contain k.
3. $\sqrt{41}\sin x\cos a + \sqrt{41}\cos x\sin a$ or $\sqrt{41}(\sin x\cos a + \cos x\sin a)$ are acceptable for $\bullet^1$ and $\bullet^3$.
4. $\bullet^2$ is not available for $k\cos x = 4$ and $k\sin x = 5$, however $\bullet^4$ may still be gained. See Candidate E.
5. $\bullet^3$ is only available for a single value of k, $k > 0$.
6. $\bullet^4$ is not available for a value of a given in degrees.
7. Accept values of a which round to 0.9.
8. Candidates may use any form of the wave function for $\bullet^1$, $\bullet^2$ and $\bullet^3$. However, $\bullet^4$ is only available if the wave is interpreted in the form $k\sin(x + a)$.
9. Evidence for $\bullet^4$ may not appear until part (b) and must appear by the $\bullet^5$ stage.

Commonly Observed Responses:

Candidate A

$\bullet^1$ ^

$\sqrt{41}\cos a = 4$

$\sqrt{41}\sin a = 5$ $\bullet^2$ ✓ $\bullet^3$ ✓

$\tan a = \frac{5}{4}$

$a = 0.896\ldots$

$\sqrt{41}\sin(x + 0.896\ldots)$ $\bullet^4$ ✓

Candidate B

$k\sin x\cos a + k\cos x\sin a$ $\bullet^1$ ✓

$\cos a = 4$

$\sin a = 5$ $\bullet^2$ ✗

$\tan a = \frac{5}{4}$

$a = 0.896\ldots$

Not consistent with equations at $\bullet^2$.

$\sqrt{41}\sin(x + 0.896\ldots)$ $\bullet^3$ ✓ $\bullet^4$ ✗

Candidate C

$\sin x\cos a + \cos x\sin a$ $\bullet^1$ ✗

$\cos a = 4$

$\sin a = 5$ $\bullet^2$ ✓ 2

$k = \sqrt{41}$ $\bullet^3$ ✓

$\tan a = \frac{5}{4}$

$a = 0.896\ldots$

$\sqrt{41}\sin(x + 0.896\ldots)$ $\bullet^4$ ✗

Question			Generic scheme	Illustrative scheme	Max mark
3.	(a)	(continued)			

Commonly Observed Responses:

Candidate D - errors at $\bullet^2$

$k\sin x\cos a + k\cos x\sin a$ $\bullet^1$ ✓

$k\cos a = 5$

$k\sin a = 4$ $\bullet^2$ ✗

$\tan a = \frac{4}{5}$

$a = 0.674\ldots$

$\sqrt{41}\sin(x+0.674\ldots)$ $\bullet^3$ ✓ $\bullet^4$ ✓ 1

Candidate E - use of x at $\bullet^2$

$k\sin x\cos a + k\cos x\sin a$ $\bullet^1$ ✓

$k\cos x = 4$

$k\sin x = 5$ $\bullet^2$ ✗

$\tan x = \frac{5}{4}$

$x = 0.896\ldots$

$\sqrt{41}\sin(x+0.896\ldots)$ $\bullet^3$ ✓ $\bullet^4$ ✓ 1

Candidate F

$k\sin A\cos B + k\cos A\sin B$ $\bullet^1$ ✗

$k\cos A = 4$

$k\sin A = 5$ $\bullet^2$ ✗

$\tan A = \frac{5}{4}$

$A = 0.896\ldots$

$\sqrt{41}\sin(x+0.896\ldots)$ $\bullet^3$ ✓ $\bullet^4$ ✓ 1

	(b)		$\bullet^5$ link to (a) $\bullet^6$ solve for $(x+a)$ $\bullet^7$ solve for x	$\bullet^5$ $\sqrt{41}\sin(x+0.896\ldots) = 5.5$ $\bullet^6$ 1.033..., 2.108... $\bullet^7$ 0.137..., 1.212...	3

Notes:

10. In part (b), where candidates work in degrees throughout, the maximum mark available is 2/3.
11. $\bullet^7$ is only available for two solutions within the stated range. Ignore 'solutions' outwith the range.
12. At $\bullet^7$ accept values of x which round to 0.1 or 1.2

Commonly Observed Responses:

Candidate G - converting to radians

⋮ $\bullet^1$ ✓ $\bullet^2$ ✓ $\bullet^3$ ✓

$\sqrt{41}\sin(x+51.3\ldots)$ $\bullet^4$ ✗

$\sqrt{41}\sin(x+51.3\ldots) = 5.5$ $\bullet^5$ ✓ 1

$x+51.3\ldots = 59.1\ldots, 120.8\ldots$

$x = 7.8\ldots, 69.4\ldots$ $\bullet^6$ ✓ 1

$x = \frac{7.9\pi}{180}, \frac{69.5\pi}{180}$ $\bullet^7$ ✓ 1

Candidate H - working in degrees and truncation

⋮ $\bullet^1$ ✓ $\bullet^2$ ✓ $\bullet^3$ ✓

$\sqrt{41}\sin(x+51.3)$ $\bullet^4$ ✗

$\sqrt{41}\sin(x+51.3) = 5.5$ $\bullet^5$ ✓ 1

$x+51.3 = 59.1, 120.9$

$x = 7.8, 69.6$ $\bullet^6$ ✓ 1 $\bullet^7$ ^

Candidate I - working in degrees

⋮ $\bullet^1$ ✓ $\bullet^2$ ✓ $\bullet^3$ ✓

$\sqrt{41}\sin(x+51.3\ldots)$ $\bullet^4$ ✗

$\sqrt{41}\sin(x+51.3\ldots) = 5.5$ $\bullet^5$ ✓ 1

$x+51.3\ldots = 59.1\ldots$

$x = 7.8\ldots$ $\bullet^6$ ^ $\bullet^7$ ^

Candidate J - working in degrees

⋮ $\bullet^1$ ✓ $\bullet^2$ ✓ $\bullet^3$ ✓

$\sqrt{41}\sin(x+51.3\ldots)$ $\bullet^4$ ✗

$\sqrt{41}\sin(x+51.3\ldots) = 5.5$ $\bullet^5$ ✓ 1

$x+51.3\ldots = 59.1\ldots, 120.8\ldots$ $\bullet^6$ ^ $\bullet^7$ ^