

$$9. \cos 2x - 5\cos x + 3 = 0$$

$$2\cos^2 x - 1 - 5\cos x + 3 = 0$$

$$2\cos^2 x - 5\cos x + 2 = 0$$

$$(2\cos x - 1)(\cos x - 2)$$

$$\cos x = \frac{1}{2} \quad \cos x = 2$$

no solutions

$$x = 60$$

$$x = 360 - 60 = 300$$

$$\underline{\underline{x = 60^\circ, 300^\circ}}$$

S	A
T	C
	360°

Question			Generic Scheme	Illustrative Scheme	Max Mar
9.			<ul style="list-style-type: none"> <li>•<sup>1</sup> substitute for <math>\cos 2x^\circ</math> into equation</li> <li>•<sup>2</sup> express in standard quadratic form</li> <li>•<sup>3</sup> factorise</li> <li>•<sup>4</sup> solve for <math>\cos x^\circ</math></li> <li>•<sup>5</sup> solve for <math>x</math></li> </ul>	<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>2\cos^2 x^\circ - 1 \dots</math></li> <li>•<sup>2</sup> <math>2\cos^2 x^\circ - 5\cos x^\circ + 2 = 0</math></li> <li>•<sup>3</sup> <math>(2\cos x^\circ - 1)(\cos x^\circ - 2) = 0</math></li> <li> <div style="display: flex; justify-content: space-around; align-items: center;"> <div style="text-align: center;"> <ul style="list-style-type: none"> <li>•<sup>4</sup></li> <li>•<sup>4</sup> <math>\cos x^\circ = \frac{1}{2}</math></li> </ul> </div> <div style="text-align: center;"> <ul style="list-style-type: none"> <li>•<sup>5</sup></li> <li><math>\cos x^\circ = 2</math></li> </ul> </div> </div> </li> <li>•<sup>5</sup> <math>x = 60, 300</math> 'no solutions'</li> </ul>	5

#### Notes:

1. •<sup>1</sup> is not available for simply stating  $\cos 2x^\circ = 2\cos^2 x^\circ - 1$  with no further working.
2. In the event of  $\cos^2 x^\circ - \sin^2 x^\circ$  or  $1 - 2\sin^2 x^\circ$  being substituted for  $\cos 2x^\circ$ , •<sup>1</sup> cannot be awarded until the equation reduces to a quadratic in  $\cos x^\circ$ .
3. Substituting  $2\cos^2 A - 1$  or  $2\cos^2 \alpha - 1$  for  $\cos 2x^\circ$  at the •<sup>1</sup> stage should be treated as bad form provided the equation is written in terms of  $x$  at •<sup>2</sup> stage. Otherwise, •<sup>1</sup> is not available.
4. Do not penalise the omission of degree signs.
5. '= 0' must appear by •<sup>3</sup> stage for •<sup>2</sup> to be awarded. However, for candidates using the quadratic formula to solve the equation, '= 0' must appear at •<sup>2</sup> stage for •<sup>2</sup> to be awarded.
6.  $\cos x^\circ = \frac{5 \pm \sqrt{9}}{4}$  gains •<sup>3</sup>.
7. Candidates may express the equation obtained at •<sup>2</sup> in the form  $2c^2 - 5c + 2 = 0$  or  $2x^2 - 5x + 2 = 0$ . In these cases, award •<sup>3</sup> for  $(2c - 1)(c - 2) = 0$  or  $(2x - 1)(x - 2) = 0$ . However, •<sup>4</sup> is only available if  $\cos x^\circ$  appears explicitly at this stage. See Candidate A.
8. The equation  $2 + 2\cos^2 x^\circ - 5\cos x^\circ = 0$  does not gain •<sup>2</sup> unless •<sup>3</sup> has been awarded.
9. •<sup>4</sup> and •<sup>5</sup> are only available as a consequence of trying to solve a quadratic equation. See Candidate B. However, •<sup>5</sup> is not available if the quadratic equation has repeated roots.
10. •<sup>3</sup>, •<sup>4</sup> and •<sup>5</sup> are not available for any attempt to solve a quadratic equation written in the form  $ax^2 + bx = c$ . See Candidate C.
11. •<sup>5</sup> is only available for 2 valid solutions within the stated range. Ignore 'solutions' outwith the range. However, see Candidate E.
12. Accept  ~~$\cos x^\circ = 2$~~  for •<sup>5</sup>. See Candidate A.

Question	Generic Scheme	Illustrative Scheme	Max Mark
9.	(continued)		
Commonly Observed Responses:			
<b>Candidate A</b> $2 \cos^2 x^\circ - 1 = 5 \cos x^\circ - 3$ $2c^2 - 5c + 2 = 0$ $(2c - 1)(c - 2) = 0$ $c = \frac{1}{2}, c = 2$ $x = 60, 300$ <del><math>\cos x^\circ = 2</math></del>		<b>Candidate B - not solving a quadratic</b> $2 \cos^2 x^\circ - 1 = 5 \cos x^\circ - 3$ $2 \cos^2 x^\circ - 5 \cos x^\circ + 2 = 0$ $-3 \cos x^\circ + 2 = 0$ $\cos x^\circ = \frac{2}{3}$	
<div>•<sup>1</sup> ✓</div> <div>•<sup>2</sup> ✓</div> <div>•<sup>3</sup> ✓</div> <div>•<sup>4</sup> ✗</div> <div>•<sup>5</sup> ✓ 1</div>		<div>•<sup>1</sup> ✓</div> <div>•<sup>2</sup> ✓</div> <div>•<sup>3</sup> ✗</div> <div>•<sup>4</sup> ✓ 2</div> <div>•<sup>5</sup> ^</div>	
<b>Candidate C - not in standard quadratic form</b> $2 \cos^2 x^\circ - 1 = 5 \cos x^\circ - 3$ $2 \cos^2 x^\circ - 5 \cos x^\circ = -2$ $\cos x^\circ (2 \cos x^\circ - 5) = -2$ $\cos x^\circ = -2, 2 \cos x^\circ - 5 = -2$ $\Rightarrow \cos x = \frac{3}{2}$ No solutions		<b>Candidate D - reading <math>\cos 2x^\circ</math> as <math>\cos^2 x^\circ</math></b> $\cos^2 x^\circ = 5 \cos x^\circ - 3$ $\cos^2 x^\circ - 5 \cos x^\circ + 3 = 0$ $\cos x^\circ = \frac{5 \pm \sqrt{13}}{2}$	
<div>•<sup>1</sup> ✓</div> <div>•<sup>2</sup> ✓ 2</div> <div>•<sup>3</sup> ✓ 2</div> <div>•<sup>4</sup> ✗</div> <div>•<sup>5</sup> ✗</div>		<div>•<sup>1</sup> ✗</div> <div>•<sup>2</sup> ✓ 1</div> <div>•<sup>3</sup> ✓ 1</div> <div>•<sup>4</sup> ^</div> <div>•<sup>5</sup> ^</div>	
<b>Candidate E</b> : $(\cos x^\circ - 1)(\cos x^\circ - 2) = 0$ $\cos x^\circ = 1, \cos x^\circ = 2$ $x = 0$ No solutions			
<div>•<sup>1</sup> ✓</div> <div>•<sup>2</sup> ✓</div> <div>•<sup>3</sup> ✗</div> <div>•<sup>4</sup> ✓ 1</div> <div>•<sup>5</sup> ✓ 1</div>			