S 2019 H2 Q4

(a) $u_{n+1} = u_n + 1$

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(a) $u_{n+1} = a u_n + b$

$u_{n+1} = \frac{98.3}{100} u_n + 30$ $\quad a = \frac{98.3}{100}, \; b = 30$

(b) (i) a is between 0 and 1, so u_{n+1} tends to limi

(ii)

$u_{n+1} = u_n$ in the limit, $= u_L$

$u_L = \frac{98.3}{100} u_L + 30$

$u_L \left(1 - \frac{98.3}{100}\right) = 30$

$u_L \left(\frac{2.7}{100}\right) = 30$

$u_L = \frac{30 \times 100}{2.7} = 1100$ mice

Question			Generic scheme	Illustrative scheme	Max mark
4.	(a)		•1 state values of a and b	•1 $a = 0\cdot 973,\ b = 30$	**1**

Notes:

1. Accept $u_{n+1} = 0\cdot 973u_n + 30$ for •1.

Commonly Observed Responses:

	(b)	(i)	•2 communicate condition for limit to exist	•2 a limit exists as the recurrence relation is linear and $-1 < 0\cdot 973 < 1$	**1**
		(ii)	•3 know how to find limit •4 process limit and state estimated population	•3 $L = 0\cdot 973L + 30$ or $L = \dfrac{30}{1-0\cdot 973}$ •4 1100	**2**

Notes:

2. For •2 accept:
   $-1 < 0\cdot 973 < 1$ or $|0\cdot 973| < 1$ or $0 < 0\cdot 973 < 1$ with no further comment;
   or statements such as "$0\cdot 973$ lies between -1 and 1";
   or $-1 < a < 1$ (as a is previously defined).
3. •2 is not available for:
   $-1 \le 0\cdot 973 \le 1$ or $0\cdot 973 < 1$;
   or statements such as "it is between -1 and 1"
4. Do not accept $L = \dfrac{b}{1-a}$ with no further working for •3.
5. For $L = 1100$ with no working award •3 and •4.

Commonly Observed Responses:

Candidate A – no rounding required

$u_{n+1} = 0\cdot 97u_n + 30$ •1 ×

$\vdots$

$L = \dfrac{30}{1-0\cdot 97}$ •3 ✓ 1

$L = 1000$ •4 ✓ 2

Candidate B – correct rounding

$u_{n+1} = 0\cdot 027u_n + 30$ •1 ×

$\vdots$

$L = \dfrac{30}{1-0\cdot 027}$ •3 ✓ 1

$L = 0$ •4 ✓ 1

Candidate C – no valid limit

$u_{n+1} = 2\cdot 7u_n + 30$ •1 ×

A limit does not exist as $2\cdot 7 > 1$ •2 ×

$L = \dfrac{30}{1-2\cdot 7}$ •3 ✓ 1

$L = 0$ •4 ×