

15

$$(a) m_{cp} = \frac{13-12}{5-8} \\ = -\frac{1}{3}$$

$$m_{\perp} = 3$$

$$y-13 = 3(x-5)$$

$$y-13 = 3x-15$$

$$y = 3x-2$$

$$(b)(i) T(0, -2)$$

$$y = 3 \times 0 - 2 = -2$$

(ii) CT is the diameter.

$$\bullet M_{CT} = \left(\frac{8+0}{2}, \frac{12+(-2)}{2} \right) = (4, 5)$$

$$\bullet d_{MC} = \sqrt{(4-8)^2 + (5-12)^2} \\ = \sqrt{16 + 49} \\ = \sqrt{65}$$

$$(x-4)^2 + (y-5)^2 = 65$$

C 2019 H2 Q15

(a) $m_{\text{radius}} = -\frac{1}{3}$, $m_{\text{TP}} = 3$

For TP,

$$\frac{y-13}{x-5} = 3$$

$$y-13 = 3x-15$$

$$y = 3x-2$$

(b)

When $\frac{x}{y} = 0$, $y = -2$
T is point $(0, -2)$

(c) $x^2 + y^2 + 2gx + 2fy + c = 0$

$$25 + 169 + 10g + 26f + c = 0$$

$$64 + 144 + 16g + 24f + c = 0$$

$$0 + 4 + 0 - 4f + c = 0$$

$$10g + 26f + c = -194$$

$$16g + 24f + c = -208$$

$$-4f + c = -4$$

$$c = 4f - 4$$

$$\begin{cases} 10g + 26f + 4f - 4 = -194 \\ 16g + 24f + 4f - 4 = -208 \end{cases}$$

$$10g + 30f = -190$$

$$16g + 28f = -204 \quad \text{--- (1)}$$

$$16g + 28f = -204 \quad \text{--- (2)}$$

$$80g + 240f = -190.8 = -1520$$

$$80g + 140f = -204.5 = -1020$$

$$100f = -500$$

$$\underline{f = -5}$$

$$c = 4f - 4 = -20 - 4$$

$$\underline{c = -24}$$

$$10g + 30f = -190$$

$$10g - 150 = -190$$

$$10g = -40$$

$$\underline{g = -4}$$

$$\underline{x^2 + y^2 - 8x - 10y + 24 = 0}$$

Or, note that $\widehat{TPC} = 90^\circ$
 \Rightarrow TC diameter.

Find mid point to get radius,
and $(x-a)^2 + (y-b)^2 = r^2$

Question			Generic scheme	Illustrative scheme	Max mark
15.	(a)		<ul style="list-style-type: none"> •¹ find gradient of radius •² state gradient of tangent •³ state equation of tangent 	<ul style="list-style-type: none"> •¹ $-\frac{1}{3}$ •² 3 •³ $y = 3x - 2$ 	3
Notes:					
<p>1. Do not accept $y = \frac{3}{1}x - 2$ for •³.</p> <p>2. •³ is only available as a consequence of trying to find and use a perpendicular gradient.</p> <p>3. At •³ accept, $y - 3x + 2 = 0$ or any other rearrangement of the equation where the constant terms have been simplified.</p>					
Commonly Observed Responses:					
	(b)	(i)	• ⁴ find coordinates of T	• ⁴ (0, -2)	1
		(ii)	<ul style="list-style-type: none"> •⁵ find midpoint CT •⁶ find radius of circle with diameter CT •⁷ state equation of circle 	<ul style="list-style-type: none"> •⁵ (4, 5) •⁶ $\sqrt{65}$ stated or implied by •⁷ •⁷ $(x - 4)^2 + (y - 5)^2 = 65$ 	3
Notes:					
<p>4. Answers in part (b)(i) must be consistent with answers from part (a).</p> <p>5. Accept $x = 0, y = -2$ for •⁴.</p> <p>6. $(x - 4)^2 + (y - 5)^2 = (\sqrt{65})^2$ does not gain •⁷.</p> <p>7. •⁷ is not available to candidates who use a line other than CT as the diameter of the circle.</p>					
Commonly Observed Responses:					

[END OF MARKING INSTRUCTIONS]