(a) Surface Area =
$$4(3x \times h) + 2(3x \times 3x - x^2) + 4(x \times h)$$

= $12xh + 16x^2 + 4xh$
= $16xh + 16x^2$

$$V = 2000$$

$$= 16x \left(\frac{2000}{8x^2}\right) + 16x^2$$

$$= 16x \left(\frac{2000}{8x^2}\right) + 16x^2$$

$$= 32000x + 16x^2$$

$$= 2000$$

$$= 8x^2 h = 2000$$

$$= 4000 + 16x^2$$

$$h = \frac{2000}{8x^2}$$

$$A = 16x^2 + \frac{4000}{x}$$

(b)
$$A = 16x^{2} + 4000x^{-1}$$
 $32x - 4000x^{-2} = 0$
 $32x - \frac{4000}{x^{2}} = 0$
 $32x^{3} - 4000 = 0$
 $32x^{3} = 125$
 $x^{3} = 5$

$$\frac{x \rightarrow 5 \rightarrow}{A'(x) - 0 +}$$
 Minimum value of A occurs when $x = 5$,

 $A'(x) - 0 +$
 $A = 16(5^2) + \frac{4000}{5}$
 $= 1200 \text{ cm}^2$

Question			Generic scheme	Illustrative scheme	Max mark
11.	(a)		•1 express A in terms of x and h	$\bullet^1 (A=)16x^2+16xh$	3
			•² express height in terms of x	$\bullet^2 h = \frac{2000}{8x^2}$	
			$ullet^3$ substitute for h and complete proof	•3 $A = 16x^2 + 16x \times \frac{2000}{8x^2}$ leading to $A = 16x^2 + \frac{4000}{x}$	

Notes:

- 1. At \bullet^1 accept any unsimplified form of $16x^2 + 16xh$.
- 2. The substitution for h at $ullet^3$ must be clearly shown for $ullet^3$ to be available.
- 3. For candidates who omit some of the surfaces of the box, only \bullet^2 is available.

Commonly Observed Responses:

1			
(b)	$ullet^4$ express A in differentiable form	$\bullet^4 16x^2 + 4000x^{-1}$	6
	• ⁵ differentiate	•5 $32x - 4000x^{-2}$	
	•6 equate expression for derivative to 0	$\bullet^6 32x - 4000x^{-2} = 0$	
	•7 process for x	•7 5	
	•8 verify nature	•8 table of signs for a derivative (see below) ∴ minimum	
		or $A''(x) = 96 > 0 \implies minimum$	
	\bullet^9 evaluate A	• 9 $A = 1200$ or min value = 1200	

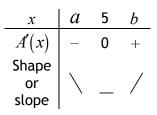
Notes:

- 4. For a numerical approach award 0/6.
- 5. •6 can be awarded for $32x = 4000x^{-2}$.
- 6. For candidates who integrate any term at the ●5 stage, only ●6 is available on follow through for setting their 'derivative' to 0.
- 7. •7, •8 and •9 are only available for working with a derivative which contains an index ≤ -2 .
- must be simplified at \bullet^7 or \bullet^8 for \bullet^7 to be awarded.
- 9. •8 is not available to candidates who consider a value of $x \le 0$ in the neighbourhood of 5.
- 10. •9 is still available in cases where a candidate's table of signs does not lead legitimately to a minimum at \bullet ⁸.
- 11. ●8 and ●9 are not available to candidates who state that the minimum exists at a negative value of X . See Candidates C and D.

For the table of signs for a derivative, accept:

$$egin{array}{c|cccc} x & 5^- & 5^+ \\ \hline A'(x) & - & 0 & + \\ \hline Shape & & & / \\ slope & & & / \\ \hline \end{array}$$

\rightarrow
) +
,
_ /



Arrows are taken to mean 'in the neighbourhood of'

Where 0 < a < 5 and b > 5

- For this question do not penalise the omission of 'x' or the word 'shape'/'slope'.
- Stating values of A'(x) in the table is an acceptable alternative to writing '+' or '-' signs. Values must be checked for accuracy.
- The only acceptable variations of A'(x) are: A', $\frac{dA}{dx}$ and $32x 4000x^{-2}$.

Commonly Observed Responses:

Candidate A - differentiating over multiple lines

$$A = 16x^{2} + 4000x^{-1}$$
$$A'(x) = 32x + 4000x^{-1}$$

$$A'(x) = 32x + 4000x^{-1}$$

 $A'(x) = 32x - 4000x^{-2}$

$$A'(x) = 32x - 4000x^{-2}$$

$$32x - 4000x^{-2} = 0$$

$$32x - 4000x^{-2} = 0$$

Candidate C - only considers 5

$$A = 16x^{2} + 4000x^{-1}$$

 $A' = 32x - 4000x^{-2} = 0$

$$x = \pm 5$$

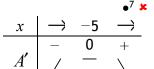
- A = 1200 or min value = 1200

Candidate D - considers 5 and negative 5 in **separate** tables

Candidate B - differentiating over multiple lines

$$A = 16x^2 + 4000x^{-1}$$

$$A' = 32x - 4000x^{-2} = 0$$
$$x = \pm 5$$



 \therefore minimum when x = 5

A = 1200 or min value = 1200

Ignore incorrect working in second table