11 (a)

Sides / Top & Bottom / insides

$$\text{Surface Area} = 4(3x \times h) + 2(3x \times 3x - x^2) + 4(x \times h)$$

$$= 12xh + 16x^2 + 4xh$$

$$= 16xh + 16x^2$$

$$V = 2000$$

Box / space

$$lbh - lbh = 2000$$

$$(3x \times 3x \times h) - (x \times x \times h) = 2000$$

$$8x^2 h = 2000$$

$$\boxed{h = \frac{2000}{8x^2}}$$

$$= 16x\left(\frac{2000}{8x^2}\right) + 16x^2$$

$$= \frac{32000x}{8x^2} + 16x^2$$

$$= \frac{4000}{x} + 16x^2$$

$$A = 16x^2 + \frac{4000}{x}$$

(b) $A = 16x^2 + 4000x^{-1}$

$$32x - 4000x^{-2} = 0$$

$$32x - \frac{4000}{x^2} = 0$$

$$32x^3 - 4000 = 0$$

$$32x^3 = 4000$$

$$x^3 = 125$$

$$x = 5$$

x	→ 5 →
$A'(x)$	− 0 +
Shape	\ _ /

Minimum value of A occurs when $x = 5$,

$$A = 16(5^2) + \frac{4000}{5}$$

$$= 1200 \text{ cm}^2$$

Question			Generic scheme	Illustrative scheme	Max mark
11.	(a)		•1 express A in terms of x and h •2 express height in terms of x •3 substitute for h and complete proof	•1 $(A=)16x^2+16xh$ •2 $h=\frac{2000}{8x^2}$ •3 $A=16x^2+16x\times\frac{2000}{8x^2}$ leading to $A=16x^2+\frac{4000}{x}$	**3**

Notes:

1. At •1 accept any unsimplified form of $16x^2+16xh$.
2. The substitution for h at •3 must be clearly shown for •3 to be available.
3. For candidates who omit some of the surfaces of the box, only •2 is available.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
	(b)		•4 express A in differentiable form •5 differentiate •6 equate expression for derivative to 0 •7 process for x •8 verify nature •9 evaluate A	•4 $16x^2+4000x^{-1}$ •5 $32x-4000x^{-2}$ •6 $32x-4000x^{-2}=0$ •7 5 •8 table of signs for a derivative (see below) $\therefore$ minimum or $A''(x)=96>0 \Rightarrow$ minimum •9 $A=1200$ or min value $=1200$	**6**

Notes:

4. For a numerical approach award 0/6.
5. •6 can be awarded for $32x = 4000x^{-2}$.
6. For candidates who integrate any term at the •5 stage, only •6 is available on follow through for setting their 'derivative' to 0.
7. •7, •8 and •9 are only available for working with a derivative which contains an index ≤ -2.
8. $\sqrt[3]{\frac{4000}{32}}$ must be simplified at •7 or •8 for •7 to be awarded.
9. •8 is not available to candidates who consider a value of $x \leq 0$ in the neighbourhood of 5.
10. •9 is still available in cases where a candidate's table of signs does not lead legitimately to a minimum at •8.
11. •8 and •9 are not available to candidates who state that the minimum exists at a negative value of x. See Candidates C and D.

For the table of signs for a derivative, accept:

x	5^-	5	5^+
$A'(x)$	$-$	0	$+$
Shape or slope	\	_	/

x	$\rightarrow$	5	$\rightarrow$
$A'(x)$	$-$	0	$+$
Shape or slope	\	_	/

Arrows are taken to mean 'in the neighbourhood of'

x	a	5	b
$A'(x)$	$-$	0	$+$
Shape or slope	\	_	/

Where $0 < a < 5$ and $b > 5$

- For this question do not penalise the omission of 'x' or the word 'shape'/'slope'.
- Stating values of $A'(x)$ in the table is an acceptable alternative to writing '+' or '–' signs. Values must be checked for accuracy.
- The only acceptable variations of $A'(x)$ are: A', $\frac{dA}{dx}$ and $32x - 4000x^{-2}$.

Commonly Observed Responses:

Candidate A - differentiating over multiple lines

•4 ^

$A'(x) = 32x + 4000x^{-1}$

$A'(x) = 32x - 4000x^{-2}$ •5 ✗

$32x - 4000x^{-2} = 0$ •6 ✓ 1

Candidate B - differentiating over multiple lines

$A = 16x^2 + 4000x^{-1}$ •4 ✓

$A'(x) = 32x + 4000x^{-1}$

$A'(x) = 32x - 4000x^{-2}$ •5 ✗

$32x - 4000x^{-2} = 0$ •6 ✓ 1

Candidate C - only considers 5

$A = 16x^2 + 4000x^{-1}$ •4 ✓

$A' = 32x - 4000x^{-2} = 0$ •5 ✓ •6 ✓

$x = \pm 5$ •7 ✗

x	$\rightarrow$	5	$\rightarrow$
A'	$-$	0	$+$
	\	_	/

$\therefore$ minimum •8 ✓ 1

$A = 1200$ or min value $= 1200$ •9 ✓ 1

Candidate D - considers 5 and negative 5 in **separate** tables

$A = 16x^2 + 4000x^{-1}$ •4 ✓

$A' = 32x - 4000x^{-2} = 0$ •5 ✓ •6 ✓

$x = \pm 5$ •7 ✗

x	$\rightarrow$	5	$\rightarrow$
A'	$-$	0	$+$
	\	_	/

x	$\rightarrow$	-5	$\rightarrow$
A'	$-$	0	$+$
	/	—	\

Ignore incorrect working in second table

$\therefore$ minimum when $x = 5$ •8 ✓ 1

$A = 1200$ or min value $= 1200$ •9 ✓ 1