P2019H2Q10

If $(x+3)$ is factor, $x=-3$ is a root.

$f(x) = 3x^4 + 10x^3 + x^2 - 8x - 6$

-3	3	10	1	-8	-6
		-9	-3	6	6
	3	1	-2	-2	0

So $Q = 3x^3 + x^2 - 2x - 2$

and $R = 0$

So $x = -3$ is a root and $(x+3)$ a factor.

$Q = 3x^3 + x^2 - 2x - 2$

Try $(x-1)$: $R = 0$

So $(x-1)$ is a factor.

$(x+3)(x-1)(3x^2+4x+2)$

1	3	1	-2	-2
		3	4	2
	3	4	2	0

$(3x+2)(x+1)$ ✗

$(3x+1)(x+2)$ ✗

Question			Generic scheme	Illustrative scheme	Max mark
10.	(a)		•¹ use −3 in synthetic division or in evaluation of quartic	•¹ $\begin{array}{r\|rrrrr} -3 & 3 & 10 & 1 & -8 & -6 \\ & 3 & & & & \end{array}$ or $3\times(-3)^4+10\times(-3)^3+(-3)^2$ $-8\times(-3)-6$	**2**
			•² complete division/evaluation and interpret result	•² $\begin{array}{r\|rrrrr} -3 & 3 & 10 & 1 & -8 & -6 \\ & & -9 & -3 & 6 & 6 \\ \hline & 3 & 1 & -2 & -2 & 0 \end{array}$ Remainder $=0\therefore(x+3)$ is a factor or $f(-3)=0\therefore(x+3)$ is a factor	

Notes:

1. Communication at •² must be consistent with working at that stage ie a candidate's working must arrive legitimately at 0 before •² can be awarded.
2. Accept any of the following for •²:
   - '$f(-3)=0$ so $(x+3)$ is a factor'
   - 'since remainder = 0, it is a factor'
   - the '0' from any method linked to the word 'factor' by 'so', 'hence', $\therefore$, $\rightarrow$, $\Rightarrow$ etc.
3. Do not accept any of the following for •²:
   - double underlining the '0' or boxing the '0' without comment
   - '$x=-3$ is a factor', '… is a root'
   - the word 'factor' only, with no link.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
	(b)				5
			•[3] identify cubic and attempt to factorise	•[3] eg $\begin{array}{r\|rrrr} \ldots & 3 & 1 & -2 & -2 \\ & & \ldots & \ldots & \ldots \\ \hline & \ldots & \ldots & \ldots & \ldots \end{array}$	
			•[4] find second factor	•[4] eg $\begin{array}{r\|rrrr} 1 & 3 & 1 & -2 & -2 \\ & & 3 & 4 & 2 \\ \hline & 3 & 4 & 2 & \boxed{0} \end{array}$ leading to $(x-1)$	
			•[5] identify quadratic	•[5] $3x^2+4x+2$	
			•[6] evaluate discriminant	•[6] -8	
			•[7] interpret discriminant and factorise fully	•[7] since $-8<0$, quadratic has no (real) factors leading to $(x+3)(x-1)(3x^2+4x+2)$	

Notes:

4. Candidates who arrive at $(x+3)(x-1)(3x^2+4x+2)$ by using algebraic long division or by inspection gain •[3], •[4] and •[5].
5. Evidence for •[6] may appear in the quadratic formula.
6. Accept '$-8<0$ so no real roots' **with the fully factorised quartic** for •[7]:
7. Do not accept any of the following for •[7]:
   - $(x+3)(x-1)(3x^2+4x+2)$ does not factorise
   - $(x+3)(x-1)(\ldots\ \ldots)(\ldots\ \ldots)$ cannot factorise further.
8. Accept $(x+3)(x-1)3x^2+4x+2$, with a valid reason for •[7].
9. Where the quadratic factor obtained at •[5] can be factorised, •[6] and •[7] are not available.

Commonly Observed Responses:

Candidate A		**Candidate B**	
$(x+3)(x-1)(3x^2+4x+2)$	•[5] ✓	$(x+3)(x-1)(3x^2+4x+2)$	•[5] ✓
$b^2-4ac=16-24<0$	•[6] ^	$b^2-4ac<0$	•[6] ^
so does not factorise	•[7] ✓ 1	so does not factorise	•[7] ^