(11) $\int_0^{\frac{\pi}{9}} \cos\left(3x - \frac{\pi}{6}\right) dx$

$= \left[\sin\left(3x - \frac{\pi}{6}\right) \times \frac{1}{3} \right]_0^{\frac{\pi}{9}}$

$= \left(\frac{1}{3} \sin\left(3\left(\frac{\pi}{9}\right) - \frac{\pi}{6}\right)\right) - \left(\frac{1}{3} \sin\left(3(0) - \frac{\pi}{6}\right)\right)$

$= \left(\frac{1}{3} \times \sin\left(\frac{\pi}{3} - \frac{\pi}{6}\right)\right) - \left(\frac{1}{3} \sin\left(-\frac{\pi}{6}\right)\right)$

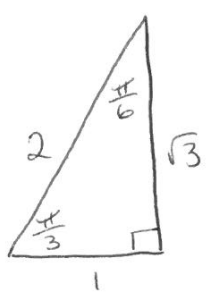
$= \left(\frac{1}{3} \sin\left(\frac{\pi}{6}\right)\right) - \left(\frac{1}{3} \sin\left(-\frac{\pi}{6}\right)\right)$

$= \left(\frac{1}{3} \times \frac{1}{2}\right) - \left(\frac{1}{3} \times \left(-\frac{1}{2}\right)\right)$

$= \frac{1}{6} - \left(-\frac{1}{6}\right)$

$= \frac{2}{6}$

$= \frac{1}{3}$



Question	Generic scheme	Illustrative scheme	Max mark
11.	•1 start to integrate	•1 $\sin\left(3x-\frac{\pi}{6}\right)\ldots$	**4**
	•2 complete integration	•2 $\ldots\times\frac{1}{3}$	
	•3 substitute limits	•3 $\left(\frac{1}{3}\sin\left(3\times\frac{\pi}{9}-\frac{\pi}{6}\right)\right)$ $-\left(\frac{1}{3}\sin\left(3\times0-\frac{\pi}{6}\right)\right)$	
	•4 evaluate integral	•4 $\frac{1}{3}$	

Notes:

1. Where candidates make no attempt to integrate or start to integrate individual terms within the bracket or use another invalid approach eg $\sin\left(3x-\frac{\pi}{6}\right)^2$ or $\int\cos(3x)-\cos\left(\frac{\pi}{6}\right)dx$, award 0/4.
2. Do not penalise the inclusion of '$+c$' or the continued appearance of the integral sign after •1.
3. Candidates who work in degrees from the start cannot gain •1. However, •2, •3 and •4 are still available.
4. •1 may be awarded for the appearance of $\sin\left(3x-\frac{\pi}{6}\right)$ in the first line of working, however see Candidates B and D.
5. •4 is only available where candidates have considered both limits within a trigonometric function.
6. Where candidates use a mixture of degrees and radians, •3 is not awarded. However, •4 is still available.

Commonly Observed Responses:

Candidate A - using addition formula

$\int_0^{\frac{\pi}{9}}\left(\cos 3x\cos\frac{\pi}{6}+\sin 3x\sin\frac{\pi}{6}\right)dx$

$=\frac{1}{3}\sin 3x\times\frac{\sqrt{3}}{2}\ldots$ •1 ✓

$\ldots-\frac{1}{3}\cos 3x\times\frac{1}{2}$ •2 ✓

Candidate B - integrated over two lines

$\int_0^{\frac{\pi}{9}}\left(\cos\left(3x-\frac{\pi}{6}\right)\right)dx$

$=\sin\left(3x-\frac{\pi}{6}\right)$ •1 ✓

$=\frac{1}{3}\sin\left(3x-\frac{\pi}{6}\right)$ •2 ✗

Candidate C - integrated in part

$3\sin\left(3x-\frac{\pi}{6}\right)$ •1 ✓ •2 ✗

$3\sin\left(3\times\frac{\pi}{9}-\frac{\pi}{6}\right)-3\sin\left(0-\frac{\pi}{6}\right)$ •3 ✓ 1

3 •4 ✓ 1

Candidate D - integrated in part

$-\frac{1}{3}\sin\left(3x-\frac{\pi}{6}\right)$ •1 ✗ •2 ✓

$-\frac{1}{3}\sin\left(3\times\frac{\pi}{9}-\frac{\pi}{6}\right)+\frac{1}{3}\sin\left(0-\frac{\pi}{6}\right)$ •3 ✓ 1

$-\frac{1}{3}$ •4 ✓ 1